NIMCET – 2023

Original Paper

01. A circle touches the x–axis and also touches the circle with centre (0, 3) and radius 2. The locus of the centre of the circle is

(a) a circle (b) an ellipse (c) a parabola (d) a hyperbola

Sol. **(c)**

Let C_1 (h,k) be the center of the circle.

Circle touches the x-axis then its radius is $r_1 = k$.

Also circle touches the circle with centre C_2

(0,3) and radius $r_2 = 2$.

$\therefore$ $|C_1 C_2| = r_1 + r_2$

$\Rightarrow \sqrt{(h-0)^2 + (k-3)^2} = |k+2|$

$\Rightarrow h^2 - 10k + 5 = 0$

Change h to x and k to y

$\Rightarrow x^2 - 10y + 5 = 0$

It is a parabola.

02. A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T_2 produces 80% of total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that P (computer turns out to be defective given that it is produced in plant T_1) = 10P (computer turns out to be defective given that it is produced in plant T_2). where P(E) denotes the probability of an event E. A computer produced in the factory is randomly selected and it does not turn out to be defective. Then the probability that it is produced in plant T_2 is

(a) $\frac{36}{73}$ (b) $\frac{47}{79}$ (c) $\frac{78}{93}$ (d) $\frac{75}{83}$

Sol. **(c)**

Let x = P (computer turns out to be defective, given that it is produced in plant T_2).

$\Rightarrow$ $x = P\left(\frac{D}{T_2}\right)$ .....(i)

where, D = Defective computer.

$\therefore$ P(computer turns out to be defective given that is produced in plant T_1) = 10x

i.e., $P\left(\frac{D}{T_1}\right) = 10x$ ....(ii)

Also, $P(T_1) = \frac{20}{100}$ and $P(T_2) = \frac{80}{100}$.

Given, P (defective computer) = $\frac{7}{100}$

i.e., $P(D) = \frac{7}{100}$

Using law of total probability,

$$P(D) = P(T_1)\cdot P\left(\frac{D}{T_1}\right) + P(T_2)\cdot\left(\frac{D}{T_2}\right)$$

$$\therefore \quad \frac{7}{100} = \left(\frac{20}{100}\right)\cdot 10x + \left(\frac{80}{100}\right)\cdot x$$

$$\Rightarrow \quad 7 = (280)\,x \Rightarrow x = \frac{1}{40} \qquad \text{.....(iii)}$$

$$\therefore \quad P\left(\frac{D}{T_2}\right) = \frac{1}{40} \text{ and } P\left(\frac{D}{T_1}\right) = \frac{10}{40}$$

$$\Rightarrow \quad P\left(\frac{\bar{D}}{T_2}\right) = 1 - \frac{1}{40} = \frac{39}{40} \text{ and } P\left(\frac{\bar{D}}{T_1}\right) = \frac{30}{40} \qquad \text{.....(iv)}$$

Using Baye's theorem,

$$P\left(\frac{T_2}{\bar{D}}\right) = \frac{P(T_2)\cdot P\left(\frac{\bar{D}}{T_2}\right)}{P(T_1)\cdot P\left(\frac{\bar{D}}{T_1}\right) + P(T_2)\cdot P\left(\frac{\bar{D}}{T_2}\right)}$$

$$= \frac{\frac{80}{100}\cdot\frac{39}{40}}{\frac{20}{100}\cdot\frac{30}{40} + \frac{80}{100}\cdot\frac{39}{40}} = \frac{78}{93}$$

03. The mean of 5 observation is 5 and their variance is 124. If three of the observations are 1,2 and 6; then the mean deviation from the mean of the data is:

(a) 2.5 (b) 2.6 (c) 2.8 (d) 2.4

Sol. **(c)**

Let the two numbers be x & y.

Given, variance $\sigma^2 = 124$

Mean, $\bar{x} = 5$ and n = 5.

$$\frac{1+2+6+x+y}{5} = 5$$

x + y = 16.

$$\text{So, mean deviation} = \frac{|1-5|+|2-5|+|6-5|+|x-5|+|y-5|}{5}$$

Now we consider x, y > 5

$$\text{Mean deviation} = \frac{4+3+1+(x+y-10)}{5} = \frac{8+16-10}{5} = \frac{14}{5} = 2.8$$

04. The perimeter of a ΔABC is 6 times the arithmetic mean of the sines of its angles. If the side a is 1, then the angle A is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) π

Sol. **(a)**

Let the sides of the triangle are a, b, c.

It is given that the perimeter of a triangle ABC is 6 times the Arithmetic Mean of the sines of its angles.

$$\therefore a+b+c=6\left(\frac{\sin A+\sin B+\sin C}{3}\right)$$

$$a+b+c=2(\sin A+\sin B+\sin C).....(1)$$

From the law of sine,

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=k$$

$$\Rightarrow a=k\sin A \Rightarrow b=k\sin B \Rightarrow c=k\sin C$$

$$\therefore a+b+c=k(\sin A+\sin B+\sin C) .....(2)$$

Hence $k=2 \Rightarrow a=2\sin A \Rightarrow 1=2\sin A \Rightarrow \sin A=\frac{1}{2}$

$$A=\frac{\pi}{6}$$

05. In an examination of nine papers, a candidate has to pass in more papers than the number of papers in which he fails in order to be successful. The number of ways in which he can be unsuccessful is

(a) 255 (b) 256 (c) 128 (d) 9 × 8!

Sol. **(b)**

The candidate is unsuccessful if he fails in 9 or 8 or 7 or 6 or 5 papers.

$\therefore$ The number of ways to be unsuccessful $= {}^9C_9 + {}^9C_8 + {}^9C_7 + {}^9C_6 + {}^9C_5$

$$= {}^9C_0 + {}^9C_1 + {}^9C_2 + {}^9C_3 + {}^9C_4 = \frac{1}{2}({}^9C_0 + {}^9C_1 + ... + {}^9C_9) = \frac{1}{2}2^9 = 256$$

06. For a group of 100 candidates, the mean and standard deviation of scores were found to be 40 and 15 respectively. Later on, it was found that the scores 25 and 35 were misread as 52 and 53 respectively. Then the corrected mean and standard deviation corresponding to the corrected figures are

(a) 39.9, 14.97 (b) 39.5, 14 (c) 39.55, 14.97 (d) 40.19, 15.1

Sol. **(c)**

$$\bar{x}=40=\frac{\sum x_i}{100} \Rightarrow \sum x_i = 4000$$

$$\sum x_i = 4000-(52+53)+(25+35)=3955 \Rightarrow \text{Correct } \bar{x}=39.55$$

As from given options only (c) option is matched.

07. Consider the following frequency distribution table.

Class interval	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	180	f_1	34	180	136	f_2	50

If the total frequency is 685 & median is 42.6 then the values of f_1 and f_2 are

(a) 80, 25 (b) 83, 22 (c) 79, 26 (d) 82, 23

Sol. **(d)**

Class	Frequency	Cumulative Frequency
10 – 20	180	180
20 – 30	f_1	$180 + f_1$
30 – 40	34	$214 + f_1$
40 – 50	180	$394 + f_1$
50 – 60	136	$530 + f_1$
60 – 70	f_2	$530 + f_1 + f_2$
70 – 80	50	$580 + f_1 + f_2$

Total frequency = 685

$580 + f_1 + f_2 = 685$

$f_1 + f_2 = 105$

Median = 42.6 (given) lies in (40 – 50) interval

Class = 40 – 50.

$$\text{Median} = \ell + \frac{\frac{N}{2} - C\cdot f}{f} \times h$$

$\ell = 40, h = 10$

$C\cdot f = 214 + f_1$

$$42.6 = 40 + \frac{\frac{685}{2} - (214 + f_1)}{180} \times 10$$

$2.6 \times 18 = 342.5 - 214 - f_1$

$f_1 = 81.7 \cong 82$

$f_2 = 105 - 82 = 23$

08. If $f(x) = \lim\limits_{x\to 0} \dfrac{6^x - 3^x - 2^x + 1}{\log_e 9(1-\cos x)}$ is a real number then $\lim\limits_{x\to 0} f(x) =$

(a) 2 (b) 3 (c) $\log_e 2$ (d) $\log_e 3$

Sol. **(c)**

$$\lim_{x\to 0} \frac{(3^x - 1)(2^x - 1)}{2\log_e 3\left(2\sin^2\frac{x}{2}\right)}$$

Using $\lim\limits_{x\to 0} \dfrac{a^x - 1}{x} = \log_e a$

$$= \frac{1}{4\log_e 3}\lim_{x\to 0}\frac{\left(3^x-1\right)}{x}\frac{\left(2^x-1\right)}{x}\frac{4\left(\frac{x^2}{4}\right)}{\sin^2\frac{x}{2}}$$

$$= \frac{1}{\log_e 3}\log_e 3 \quad \log_3 2 = \log_e 2$$

09. The sum of infinite terms of decreasing GP is equal to the greatest value of the function $f(x) = x^3 + 3x - 9$ in the interval $[-2, 3]$ and difference between the first two terms is $f'(0)$. Then the common ratio of the GP is

(a) $-\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $+\frac{2}{3}$ (d) $-\frac{4}{3}$

Sol. **(c)**

$f(x) = x^3 + 3x - 9 \qquad x \in [-2, 3]$

Differentiate with respect to x

$f'(x) = 3x^2 + 3$

Hence f(x) is strictly increasing function so its greatest value will be at $x = 3$

$f(3) = 3^3 + 3 \times 3 - 9 = 27$

$$\frac{a}{1-r} = 27$$

$$\Rightarrow a = 27 - 27r$$

$$\Rightarrow a + 27r = 27 \qquad \text{.....(1)}$$

$f'(0) = 3$

Also given $a - ar = f'(0)$

$$\Rightarrow a(1-r) = 3$$

$$\Rightarrow 1 - r = \frac{3}{a} \qquad \text{.....(2)}$$

From eq, 1 and 2 we get

$$a + 27\left(1 - \frac{3}{a}\right) = 27$$

$$a + 27 - \frac{81}{a} = 27$$

$$\Rightarrow a^2 = 81 \Rightarrow a = \pm 9$$

$\because$ G.P. is decreasing

$\therefore$ a = 9.

Now, $\frac{9}{1-r} = 27 \Rightarrow \frac{1}{3} = 1 - r$

$$r = 1 - \frac{1}{3}$$

$$r = \frac{2}{3}$$

10. The value of $\int_{-\pi/3}^{\pi/3} \frac{x \sin x}{\cos^2 x} dx$ is

(a) $\frac{1}{3}(4\pi+1)$ (b) $\frac{4\pi}{3} - 2\log\tan\frac{5\pi}{12}$ (c) $\frac{4\pi}{3} + \log\tan\frac{5\pi}{12}$ (d) $\frac{4\pi}{3} - \log\tan\frac{5\pi}{3}$

Sol. **(b)**

$I = 2\int_0^{\pi/3} x\tan x\sec x dx$ Using integration by parts

$I = 2\left[x\sec x\right]_0^{\pi/3} - 2\int_0^{\pi/3} \sec x dx$

$= 2\left[\frac{\pi}{3}\times 2\right] - 2\left[\ln|\sec x + \tan x\right]_0^{\pi/3}$

$= \frac{4\pi}{3} - \left[\ln|2+\sqrt{3}|\right] = \frac{4\pi}{3} - 2\ln\tan\frac{5\pi}{12}$

11. The equation of the tangent at any point of curve $x = a\cos 2t$, $y = 2\sqrt{2}a\sin t$ with m as its slope is

(a) $y = mx + a\left(m - \frac{1}{m}\right)$ (b) $y = mx - a\left(m + \frac{1}{m}\right)$

(c) $y = mx + a\left(a + \frac{1}{a}\right)$ (d) $y = amx + a\left(m - \frac{1}{m}\right)$

Sol. **(b)**

Eq. of tangent, with slope m.

$x = a\cos 2t$ $y = 2\sqrt{2}a\sin t$

$\frac{dx}{dt} = -2a\sin 2t$ $\frac{dy}{dt} = 2\sqrt{2}a\cos t$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2\sqrt{2}a\cos t}{-2a\sin 2t} = \frac{\sqrt{2}\cos t}{-2\sin t\cos t} = \frac{-1}{\sqrt{2}\sin t} = m$ given

$\sin t = \frac{-1}{\sqrt{2}m}$

$\sin t = \frac{-1}{\sqrt{2}m} \Rightarrow \boxed{\sin t = \frac{-1}{\sqrt{2}m}}$

Then

$x = a\cos 2t$ $y = 2\sqrt{2}a\sin t$

$= a\left(1 - 2\sin^2 t\right)$ $= 2\sqrt{2}a\times\left(\frac{-1}{\sqrt{2}m}\right)$

$= a\left(1 - 2\times\left(\frac{-1}{\sqrt{2}m}\right)^2\right)$ $y = \frac{-2a}{m}$

$$x = a\left(1-\frac{1}{m^2}\right)$$

Then Eq. of tangent.

$$y - y_1 = m(x - x_1)$$

$$y + \frac{2a}{m} = m\left(x - a\left(1-\frac{1}{m^2}\right)\right)$$

$$y + \frac{2a}{m} = mx - am + \frac{a}{m}$$

$$y = mx - a\left(m + \frac{1}{m}\right)$$

12. If $\prod_{i=1}^{n} \tan(\alpha_i) = 1 \ \ \forall \alpha_i \in \left[0, \frac{\pi}{2}\right]$ where i = 1, 2, 3, ..., n. Then maximum of value of $\prod_{i=1}^{n} \sin\alpha_i$.

(a) $\frac{1}{2^n}$ (b) $\frac{1}{2^{n/2}}$ (c) 1 (d) None of these

Sol. **(b)**

$$\sin\alpha_1 \sin\alpha_2 \sin\alpha_3 .....\sin\alpha_n = \cos\alpha_1 \cos\alpha_2 ..... \cos\alpha_n$$

Multiplying both sides by $\sin\alpha_1 \sin\alpha_2 \sin\alpha_3 .....\sin\alpha_n$

$$\Rightarrow \sin^2\alpha_1 \sin^2\alpha_2 \sin^2\alpha_3 .....\sin^2\alpha_n = (\sin\alpha_1 \cos\alpha_1)(\sin\alpha_2 \cos\alpha_2) ..... (\sin\alpha_n \cos\alpha_n)$$

$$\Rightarrow \sin^2\alpha_1 \sin^2\alpha_2 \sin^2\alpha_3 .....\sin^2\alpha_n = \frac{1}{2^n}(\sin 2\alpha_1)(\sin 2\alpha_2) ..... (\sin 2\alpha_n)$$

As we know maximum value of $\sin\theta$ is 1

$$\Rightarrow \sin^2\alpha_1 \sin^2\alpha_2 \sin^2\alpha_3 .....\sin^2\alpha_n \le \frac{1}{2^n}$$

$$\Rightarrow \sin\alpha_1 \sin\alpha_2 \sin\alpha_3 .....\sin\alpha_n \le \frac{1}{2^{n/2}}$$

Hence maximum value of $\prod_{i=1}^{n} \sin\alpha_i = \frac{1}{2^{n/2}}$

13. A speaks truth in 60% and B speaks the truth in 50% cases. In what percentage of cases they are likely in contradict each other while narrating some incident is

(a) 1/2 (b) 1/4 (c) 2/3 (d) 1/3

Sol. **(a)**

Probability A speaks truth $P(AT) = \frac{60}{100} = \frac{3}{5}$

$\Rightarrow$ Probability A speaks lie $P(AL) = \frac{40}{100} = \frac{2}{5}$

Probability B speaks truth $P(BT) = \frac{50}{100} = \frac{1}{2}$

$\Rightarrow$ Probability George speaks lie $P(BL) = \frac{50}{100} = \frac{1}{2}$

Probability that they contradict each other stating the same fact = $P(AT \cap BL) + P(AL \cap BT)$

$= \frac{3}{5} \times \frac{1}{2} + \frac{2}{5} \times \frac{1}{2} = \frac{1}{2}$

14. If a and b are vector in space, given by $a = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}}$ and $b = \frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$, then the value of $(2a + b) \cdot [(a \times b) \times (a - 2b)]$ is

(a) 3 (b) 4 (c) 5 (d) 6

Sol. **(c)**

$-(2a+b) \cdot \left[(a-2b) \times (a \times b)\right]$

$= -(2a+b) - \left[\{(a-2b) \cdot b\}\vec{a} - \{(a-2b) \cdot \vec{a}\}\vec{b}\right]$

$= (2a+b)\left[\{(a-2b) \cdot a\}\vec{b} - \{(a-2b) \cdot b\}\vec{a}\right]$

$= (2a+b)\left[\{|a|^2 - 2b \cdot a\}\vec{b} - \{a \cdot b - 2|b|^2\}\vec{a}\right]$

Now, $\vec{a} \cdot \vec{b} = \left(\frac{\hat{i} - 2\hat{j}}{\sqrt{5}}\right) \cdot \left(\frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}\right) = 2 - 2 = 0$

$|\vec{a}| = 1, |\hat{b}| = 1$

$= (2a+b) \cdot \left[\{1-0\}\vec{b} - \{0-2\}\vec{a}\right]$

$= (2a+b) \cdot \left[\vec{b} + 2\vec{a}\right]$

$= (2a+b) \cdot [b + 2a]$

$= 2a \cdot b + 4|a|^2 + |b|^2 + 4\vec{a} \cdot \vec{b}$

$= 0 + 4 + 1 + 0 = 5$

15. Let A = 2i + j – 2k and B = i + j, If C is a vector such that |C – A| = 3 and the angle between A × B and C is 30°, then [(A × B) × C] = 3 then the value of $\vec{A} \cdot \vec{C}$ is equal to

(a) 25/8 (b) 2 (c) 5 (d) 1/8

Sol. **(b)**

$A \times B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix}$ $\hat{i}(2) - \hat{j}(2) + \hat{k}(1)$

$A \times B = 2\hat{i} - 2\hat{j} + \hat{k}$

$|C - A| = 3$

$|C - A|^2 = 9$

$|C|^2+|A|^2-2C\cdot A=9\Rightarrow|C|^2-2C\cdot A=0$ .....(i)

Now

$|(A\times B)\times C|=|A\times B||C|=\sqrt{4+4+1}\times|C|\times\sin 30^o=3$

Given $\theta=30^o$

$\Rightarrow|C|=2$

Put this value in equation (i)

We get $\vec{C}\cdot\vec{A}=2$

16. Let A and B be sets. $A\cap X=B\cap X=\phi$ and $A\cup X=B\cup X$ for some set X, relation between A & B.

(a) $A=B$ (b) $A\cup B=X$ (c) $B=X$ (d) $A=X$

Sol. **(a)**

Let A and B be two sets such that $A\cap X=B\cap X=\phi$ and $A\cup X=B\cup X$ for some set X.

To show: $A=B$

$A=A\cap(A\cup X)$

$=A\cap(A\cup X)\quad(A\cup X=B\cup X)$

$=(A\cap B)\cup(A\cap X)$ (Distributive law)

$(A\cap B)\cup\phi(\because A\cap X=\phi)$

$=A\cap B$ .....(i)

Now, $B=B\cap(B\cup X)$

$=B\cap(A\cup X)(\because A\cup X=B\cup X)$

$=(B\cap A)\cup(B\cap X)$ (Distributive law)

$=(B\cap A)\cup\phi(\because B\cap X=\phi)$

$=B\cap A=A\cap B$ .....(ii)

Hence, form (i) and (ii), we get

$A=B$.

17. If a, b, c, d are in HP and arithmetic mean of ab, bc, cd is 9 then which of the following number is the value of ad?

(a) 3 (b) 9 (c) 12 (d) 4

Sol. **(b)**

$\frac{ab+bc+cd}{3}=9$

$\Rightarrow ab+bc+cd=27$

a, b, c are in HP $\Rightarrow b=\frac{2ac}{a+c}\Rightarrow a+c=\frac{2ac}{b}$ .....(i)

b, c, d are in HP $\Rightarrow c=\frac{2bd}{b+d}\Rightarrow b+d=\frac{2bd}{c}$ .....(ii)

Multiply (i) and (ii)

$$(a+c)(b+d)=\frac{2ac}{b}\frac{2bd}{c}=4ad$$

$$(ab+ad+bc+dc)=4ad$$

$$\Rightarrow 3ad=ab+bc+cd=27$$

$$\Rightarrow ad=9$$

18. Find foci of the equation $x^2 + 2x - 4y^2 + 8y - 7 = 0$

(a) $\left(\sqrt{5}\pm1,1\right)$ (b) $\left(-1\pm\sqrt{5},1\right)$ (c) $\left(-1\sqrt{5}\pm1\right)$ (d) $\left(1,-1\pm\sqrt{5}\right)$

Sol. **(b)**

$$\left(x^2+2x+1\right)-4\left(y^2-2y\right)=7+1$$

$$\Rightarrow (x+1)^2-4(y-1)^2=4$$

$$\Rightarrow \frac{(x+1)^2}{4}-\frac{(y-1)^2}{1}=1$$

Hence center is (–1, 1)

$$b^2=a^2\left(e^2-1\right)\Rightarrow 1=4\left(e^2-1\right)$$

$$\Rightarrow e=\frac{\sqrt{5}}{2}\Rightarrow ae=\sqrt{5}$$

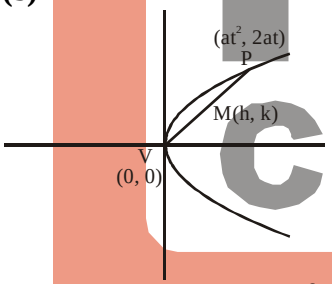
focii are at a distance ae from center.

Hence focii will be $\left(-1+\sqrt{5},1\right)\&\left(-1-\sqrt{5},1\right)$

19. The locus of the mid-point of all chords of the parabola $y^2 = 4x$ which are drawn through its vertex is

(a) $y^2 = 8x$ (b) $y^2 = 2x$ (c) $y^2 + 4y^2 = 16$ (d) $x^2 = 2y$

Sol. **(b)**



Let M be the mid point of VP is (h, k)

$$h=\frac{0+at^2}{2}\Rightarrow at^2=2h \qquad .....(1)$$

$$k=\frac{0+2at}{2}\Rightarrow at=k\Rightarrow t=\frac{k}{a}$$

Put value of t in equation (1) we get $k^2 = 2ah$

Replace $h\to x$ and $k\to y$

$y^2 = 2ax$

20. If $a=\hat{i}-\hat{k},\ b=x\hat{i}+\hat{j}+(1-x)\hat{k}$ and $c=y\hat{i}+x\hat{j}+(1+x-y)\hat{k}$, then [**a b c**] depends on

(a) Neither x nor y (b) Only x (c) Only y (d) Both x and y

Sol. **(a)**

$$\begin{vmatrix}1 & 0 & -1\\ x & 1 & 1-x\\ y & x & 1+x-y\end{vmatrix}=\left\{(1+x-y)-\left(x-x^2\right)\right\}-0-\left\{x^2-y\right\}$$

$=1+x-y-x+x^2-x^2+y$

$=1$

Depends Neither on x nor, y

21. If $\vec{a},\vec{b}$ are unit vectors such that $2\vec{a}+\vec{b}=3$ then which of the following statement is true?

(a) $\vec{a}$ is parallel to $\vec{b}$ (b) $\vec{a}$ is perpendicular to $\vec{b}$

(c) $\vec{a}$ is perpendicular to $2\vec{a}+\vec{b}$ (d) $\vec{b}$ is parallel to $2\vec{a}+\vec{b}$

Sol. **(a)**

Fundamentally this question is wrong because sum of two vectors can not be equal to scalar.
But if we have solve this question.

$$\left|2\vec{a}+\vec{b}\right|^2=9$$

$$\Rightarrow 4|\vec{a}|^2+|\vec{b}|^2+4\vec{a}\cdot\vec{b}=9$$

$$\Rightarrow \vec{a}\cdot\vec{b}=1$$

$$\Rightarrow |\vec{a}|\cdot|\vec{b}|\cos\theta=1$$

$$\Rightarrow \cos\theta=1$$

$$\Rightarrow \theta=0$$

22. $\int f(x)dx=g(x)$ then $\int x^5 f(x^3)dx$

(a) $\frac{1}{3}x^3g\left(x^3\right)-3\int x^4g\left(x^3\right)dx+c$ (b) $\frac{1}{3}x^3g\left(x^3\right)-\int x^2g\left(x^3\right)dx+c$

(c) $\frac{1}{3}x^3g\left(x^3\right)-\int x^3g\left(x^3\right)dx+c$ (d) None of these

Sol. **(b)**

Let $x^3=t$

$$\Rightarrow 3x^2dx=dt$$

$$I=\int x^5f(x^3)dx$$

$$=\int x^2x^3f(x^3)dx$$

$$=\frac{1}{3}\int tf(t)dt$$

Using integration by parts

$$I = \frac{1}{3}\left[t\int f(t)dt - \int\left(\frac{dt}{dt}\int f(t)dt\right)dt \right]$$

$$= \frac{1}{3}\left[tg(t) - \int g(t)dt \right]$$

$$= \frac{1}{3}tg(t) - \frac{1}{3}\int g(t)dt$$

As $x^3 = t$ and $dt = 3x^2\,dx$

$$= \frac{1}{3}x^3 g(x^3) - \frac{3}{3}\int x^2 g(x^3)dx$$

$$= \frac{1}{3}x^3 g(x^3) - \int x^2 g(x^3)dx + c$$

23. $\lim\limits_{x\to 1}\dfrac{x^4-1}{x-1} = \lim\limits_{x\to k}\dfrac{x^3-k^2}{x^2-k^2}$ then find k

(a) 8/3 (b) 4/3 (c) 2/3 (d) 1

Sol. **(a)**

Using L'Hospital rule

$$\lim_{x\to 1}\frac{4x^3}{1} = \lim_{x\to k}\frac{3x^2}{2x}$$

$$\Rightarrow 4 = \frac{3}{2}k$$

$$k = \frac{8}{3}$$

24. The graph of function $f(x) = \log_e\left(x^3 + \sqrt{x^6+1}\right)$ is symmetric about:

(a) x–axis (b) y-axis (c) origin (d) y = x

Sol. **(c)**

$$f(x) = \log\left(x^3 + \sqrt{x^6+1}\right)$$

$$f(-x) = \log\left[(-x)^3 + \sqrt{(-x)^6+1}\right]$$

$$f(-x) = \log\left[\sqrt{x^6+1} - x^3\right]$$

$$= \log\left[\frac{\left(\sqrt{x^6+1}-x^3\right)\left(\sqrt{x^6+1}+x^3\right)}{\sqrt{x^6+1}+x^3}\right]$$

$$= \log\left(\frac{1}{x^3+\sqrt{x^6+1}}\right)$$

$= \log\left(x^3 + \sqrt{x^6+1}\right)^{-1}$

$= -\log\left(x^3 + \sqrt{x^6+1}\right)$

$= -f(x)$ Odd function

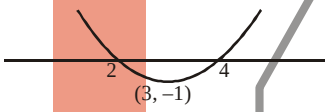
We should know that odd functions are symmetrical about origin.

25. If the equation $|x^2 - 6x + 8| = a$ has four real solution then find the value of a?

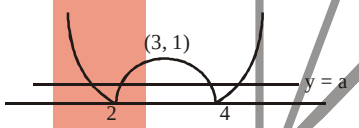
(a) $a \in 0$ (b) $a = 1$ (c) $a \in (0,1)$ (d) $a \in [1,2]$

Sol. **(c)**

Let $y = x^2 - 6x + 8$



$y = |x^2 - 6x + 8|$



Hence for 4 solutions a must lie between (0, 1).

26. Largest value of $\cos^2\theta - 6\sin\theta\cos\theta + 3\sin^2\theta + 2$

(a) 4 (b) 0 (c) $4+\sqrt{10}$ (d) $4-\sqrt{10}$

Sol. **(c)**

$\cos^2\theta - 6\sin\theta\cos\theta + 3\sin^2\theta + 2$

$2\sin^2\theta - 6\sin\theta\cos\theta + 3$

$3 - 3\sin 2\theta + [1 - \cos 2\theta]$

$= 4 - [3\sin 2\theta + \cos 2\theta]$

$-\sqrt{10} \le 3\sin 2\theta + \cos 2\theta \le \sqrt{10}$

For maximum value of given expression

$3\sin 2\theta + \cos 2\theta$

It should be minimum. Hence maximum value is $4+\sqrt{10}$

27. Given to events A and B such that odd in favour A are 2 : 1 and odd in favour of $A \cup B$ are 3 : 1. Consistent with this information the smallest and largest value for the probability of event B are given by

(a) $\frac{1}{12} \le P(B) \le \frac{3}{4}$ (b) $\frac{1}{3} \le P(B) \le \frac{1}{2}$ (c) $\frac{1}{6} \le P(B) \le \frac{1}{3}$ (d) None of these

Sol. **(a)**

$P(A) = \frac{2}{3}$

$P(A \cup B) = \frac{3}{4}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{3}{4} = \frac{2}{3} + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(B) - \frac{1}{12}$$

$$0 \le P(A \cap B) \le P(A)$$

$$0 \le P(B) - \frac{1}{12} \le \frac{2}{3}$$

$$\frac{1}{12} \le P(B) \le \frac{2}{3} + \frac{1}{12}$$

$$\frac{1}{12} \le P(B) \le \frac{3}{4}$$

28. If A and B are square matrices such that $B = -A^{-1}BA$, then $(A + B)^2$ is

(a) 0 (b) $A^2 + B^2$ (c) $A^2 + 2AB + B^2$ (d) $A + B$

Sol. **(b)**

$$B = -A^{-1}BA$$

$$AB = -\left(AA^{-1}\right)BA$$

$$AB = -BA$$

$$\left(A + B\right)^2 = A^2 + B^2 + AB + BA = A^2 + B^2$$

29. A bag contain different kind of balls in which 5 yellow, 4 black & 3 green balls. If 3 balls are drawn at random then find the probability that no black ball is chosen

(a) $\frac{14}{55}$ (b) $\frac{1}{66}$ (c) $\frac{2}{9}$ (d) None of these

Sol. **(a)**

$5Y + 4B + 3G = 12$ balls

Non black ball = 8 balls

$$P(\text{No black ball is selected}) = \frac{{}^8C_3}{{}^{12}C_3} = \frac{8 \times 7 \times 6}{12 \times 11 \times 10} = \frac{14}{55}$$

30. Between any two real roots of the equation $e^x \sin x = 1$, the equation $e^x \cos x = -1$ has

(a) Atleast one root (b) Exactly one root (c) No root (d) None of these

Sol. **(a)**

$$e^x \sin x = 1$$

$$f(x) = \sin x - e^{-x}$$

Let it has 2 real roots α, β.

$$f'(x) = \cos x + e^{-x} = 0$$

Using Rolle's theorem

There will be atleast one real root of its derivative between α and β.

atleast one $c \in (\alpha, \beta)$

$f'(c) = 0$

$e^{-c} + \cos c = 0$

$1 + e^{c} \cos c = 0 = g(c)$

$g(x) = e^{x} \cos x + 1$

$\alpha < c < b$

Atleast one real root of g(x) between two real root of f(x).

31. If f(x) is a polynomial of degree 4, f(n) = n + 1 & f(0) = 25, then find f(5) = ?

(a) 30 (b) 20 (c) 25 (d) None of these

Sol. **(a)**

$f(x) \rightarrow 4$ degree polynomial

Let

$f(x) = \lambda(x-1)(x-2)(x-3)(x-4) + (x+1)$

$f(0) = \lambda(-1)(-2)(-3)(-4) + 1 = 25$

$\Rightarrow \lambda = 1$

$f(5) = 6 + 24\lambda$ Put $\lambda = 1$

f(5) = 30.

32. The maximum value of $f(x) = (x-1)^2 (x+1)^3$ is equal to $\frac{2^p 3^q}{3125}$ then the ordered pair of (p, q) will be

(a) (3, 7) (b) (7, 3) (c) (5, 5) (d) (4, 4)

Sol. **(b)**

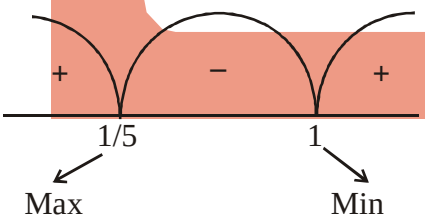
$f(x) = (x+1)^2 (x+1)^3$

$f'(x) = 2(x-1)(x+1)^3 + 3(x+1)^2 (x-1)^2$

$= (x-1)(x+1)^2 [2(x+1) + 3(x-1)] = 0$

$= (x-1)(x+1)^2 [5x-1]$

x = 1, –1, 1/5



$f\left(\frac{1}{5}\right) = \left(\frac{1}{5}-1\right)^2 \left(\frac{1}{5}+1\right)^3 = \left(-\frac{4}{5}\right)^2 \left(\frac{6}{5}\right)^3$

$= \frac{16}{5^2} \times \frac{6^3}{5^3} = \frac{2^7 \times 3^3}{5^5}$

Hence, p = 7, q = 3.

33. The coefficient of x^{50} in the expression of $(1+x)^{1000} + 2x(1+x)^{999} + 3x^2(1+x)^{998} + \ldots\ldots + 1001x^{1000}$

(a) ${}^{1005}C_{50}$ (b) ${}^{1005}C_{48}$ (c) ${}^{1002}C_{50}$ (d) ${}^{1002}C_{51}$

Sol. **(c)**

$\text{Let } S = (1+x)^{1000} + 2x(1+x)^{999} + 3x^2(1+x)^{998} + \ldots. + 1000x^{999}(1+x) + 1001x^{1000}$

Above is A.G.P. of common ratio $r = \dfrac{x}{1+x}$

$$\therefore \left[\frac{x}{(1+x)}\right] S = x(1+x)^{999} + 2x^2(1+x)^{998} + \ldots. + 1000 \cdot x^{1000} + \frac{1001x^{1001}}{1+x}$$

Subtracting, $\left(1 - \dfrac{x}{1+x}\right) S = (1+x)^{1000} + x(1+x)^{999} + x^2(1+x)^{998} + \ldots. + x^{1000} - \dfrac{1001x^{1001}}{1+x}$

Or, $S = (1+x)^{1001} + x(1+x)^{1000} + x^2(1+x)^{999} + \ldots.. + x^{1000}(1+x) - 1001x^{1001}$

$$= \frac{(1+x)^{1001}\left[1 - \left(x - (1+x)\right)^{1001}\right]}{1-x} - 1001x^{1001}$$

Sum G.P. $(1+x)^{1002}\left[1 - \left(\dfrac{x}{(1+x)}\right)^{1001}\right] - 1001x^{1001}$

$= (1+x)^{1002} - x^{1001}(1+x) - 1001x^{1001}$

$= (1+x)^{1002} - x^{1002} - 1002x^{1001}$ .....(i)

Now the coefficients of x^{50} on the R.H.S. of (i)

$= {}^{1002}C_{50}$

34. If $x_k = \cos\left(\dfrac{2\pi k}{n}\right) + i\sin\left(\dfrac{2\pi k}{n}\right)$, then $\sum_{k=1}^{n}(x_k) = ?$

(a) 1 (b) –1 (c) 0 (d) None of these

Sol. **(c)**

We should know that $\cos\theta + i\sin\theta = e^{i\theta}$

$$x_k = \frac{\cos 2\pi k}{n} + \frac{i\sin 2\pi k}{n} = e^{i\frac{2k\pi}{n}}$$

$$\sum_{k=1}^{n} x_k = \sum_{k=1}^{n} e^{i\frac{2k\pi}{n}} = e^{i\frac{2\pi}{n}} + e^{i\frac{4\pi}{n}} + e^{i\frac{6\pi}{n}} + \ldots\ldots + e^{i\frac{2n\pi}{n}}$$

Let $e^{i\frac{2\pi}{n}} = \alpha$

Hence this series $= \alpha + \alpha^2 + \alpha^3 + \ldots\ldots + \alpha^n = \dfrac{\alpha(1-\alpha^n)}{1-\alpha}$

$$= \frac{e^{i\frac{2\pi n}{n}}\left(1-e^{\left(\frac{i2\pi}{n}\right)n}\right)}{1-e^{\frac{i2\pi}{n}}}$$

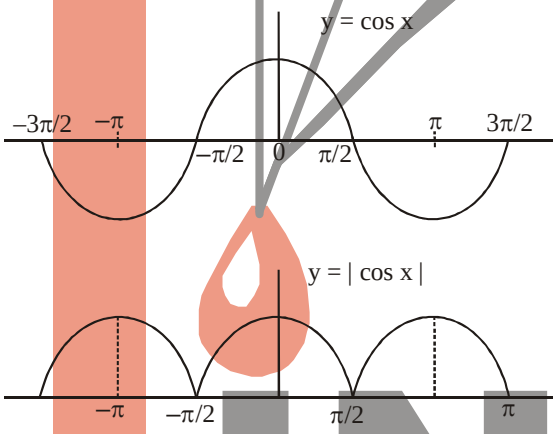
$$\left[e^{i2\pi} = \cos 2\pi + i\sin 2\pi = 1\right]$$

$$= \frac{e^{i\frac{2\pi n}{n}}\left(1-e^{i2\pi}\right)}{1-e^{\frac{i2\pi}{n}}} = 0$$

35. Number of point of which f(x) is not differentiable $f(x) = |\cos x| + 3 \text{ in } [-\pi, \pi]$

(a) 2 (b) 3 (c) 4 (d) None of these

Sol. **(a)**



It is not differentiable at two points (We should know the function is not differentiable where there is sharp turns).

$y = |\cos x| + 3$ It is not differentiable at $x = \frac{-\pi}{2}, \frac{-\pi}{2}$

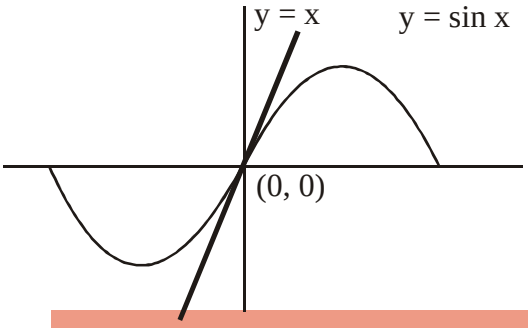
36. If n_1 and n_2 are the number of real valued solutions $x = |\sin^{-1} x|$ & $x = \sin(x)$ respectively, then the value of $n_2 - n_1$ is

(a) 1 (b) 0 (c) 2 (d) 3

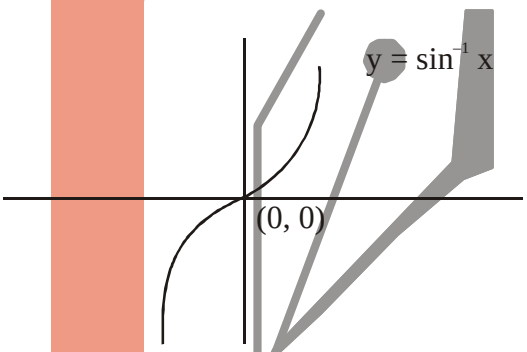
Sol. **(a)**

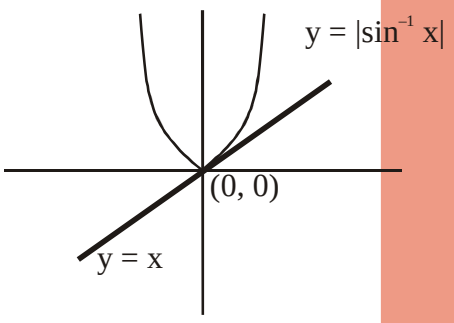
$x = \sin x$

We should know that $\sin x < x < \tan x$ when $x \in \left(0, \frac{\pi}{2}\right)$



Hence $n_2 = 1$ (It has only one solution that (0, 0))
We should know that graph of $f^{-1}(x)$ is mirror image of f(x) with respect to y = x as a mirror.





Number of solutions $n_1 = 1$ (only (0, 0) is the solution of this equation) of eq. $x = |\sin^{-1} x|$
Hence $n_2 - n_1 = 0$

37. Let a, b, c, d be no zero numbers. If the point of intersection of the line $4ax + 2ay + c = 0$ & $5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistance from the two are then

(a) $a + b + c + d = 0$ (b) $ad - bc = 0$ (c) $3bc - 2ad = 0$ (d) $3bc + 2ad = 0$

Sol. **(c)**
If it lies in the fourth quadrant, we get (x, –x)
$2ax + c = 0$ and $3bx + d = 0$

$$\frac{c}{2a} = \frac{d}{3b}$$

$3bc - 2ad = 0$

38. The negation of $\sim S \vee (\sim R \wedge S)$ is equivalent to

(a) $S \vee (R \vee -S)$ (b) $S \wedge \sim R$ (c) $S \wedge R$ (d) $S \wedge (R \wedge \sim S)$

Sol. **(c)**

$$\sim(\sim S \vee (\sim R \wedge S))$$

$$= S \wedge (R \vee \sim S)$$

$$= S \cdot (R + \overline{S}) = S \cdot R + S \cdot \overline{S}$$

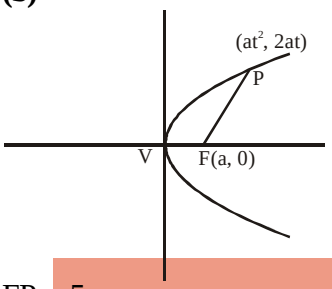
$$= S \cap R + S \cap \overline{S} = S \cdot R + \phi$$

$$= (S \cap R) \cup \phi = S \cap R$$

39. A point P in the first quadrant, lies on $y^2 = 4ax$, $a > 0$, and keeps a distance of 5a units from its focus. Which of the following points lies on the locus of P?

(a) (1, 0) (b) (1, 1) (c) (0, 2) (d) (2, 0)

Sol. **(b)**



$FP = 5a$

$a + at^2 = 5a$

$1 + t^2 = 5$

$t^2 = 4$

$t = \pm 2$

$t > 0 \Rightarrow t = 2$

Hence $P \equiv (4a, 4a)$

From given options only (1, 1) satisfy P.

40. If $\int x \sin x \sec^3 x\,dx = \frac{1}{2}\left[f(x)\sec^2 x + g(x)\left(\frac{\tan x}{x}\right)\right] + c$ then which of the following is true

(a) $f(x) - g(x) = 0$ (b) $f(x) \cdot g(x) = 0$ (c) $f(x) + g(x) = 0$ (d) $f(x) + g(x) = 1$

Sol. **(c)**

$\int x \sin x \sec^3 x\,dx = \int x \tan x \sec^2 x\,dx$ Using integration by part

$= \left[x \int \sec^2 x \tan x\,dx\right] - \int \left(\frac{dx}{dx}\int \sec^2 x \tan x\,dx\right)dx$

$= \left[x\frac{\tan^2 x}{2}\right] - \int \frac{\tan^2 x}{2}dx$

$= \frac{1}{2}\left[x\tan^2 x - \int(\sec^2 x - 1)dx\right]$

$= \frac{1}{2}\left[x\tan^2 x - \tan x + x\right] + c$

$= \frac{1}{2}\left[x(1 + \tan^2 x) - \tan x\right] + c$

$= \frac{1}{2}\left[x\sec^2 x - \tan x\right] + c$

$= \frac{1}{2}\left[f(x)\sec^2 x + g(x)\left(\frac{\tan x}{x}\right)\right] + c$

Hence, $f(x) = x : g(x) = -x$

Hence $f(x) + g(x) = 0$

41. $\theta = \cos^{-1}\left(\frac{3}{\sqrt{20}}\right)$ is the angle between $\vec{a} = \hat{i} - 2x\hat{j} + 2y\hat{k}$ & $\vec{b} = x\hat{i} + \hat{j} + y\hat{k}$ then possible values at (x, y) that lie on the locus

(a) (0, 1) (b) (1, 0) (c) (1, 1) (d) (0, 0)

Sol. **(a)**

$\vec{a} = \hat{i} - 2x\hat{j} + 2y\hat{k}$ & $\vec{b} = x\hat{i} + \hat{j} + y\hat{k}$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \cos\cos^{-1}\left(\frac{3}{\sqrt{20}}\right) = \frac{3}{\sqrt{20}}$$

$$\frac{3}{\sqrt{20}} = \frac{x - 2x + 2y^2}{\sqrt{1+4x^2+4y^2}\sqrt{x^2+1+y^2}}$$

$$\Rightarrow 3\sqrt{1+4x^2+4y^2}\sqrt{x^2+1+y^2} = \sqrt{20}\left[2y^2 - x\right]$$

From Given options (0, 1) satisfy given equations.

42. Let R be reflexive relation on the finite set a having 10 elements and if m is the number of ordered pair in R, then

(a) $m \geq 10$ (b) m = 100 (c) m = 10 (d) $m \leq 10$

Sol. **(a)**

Given R has m order pairs.

Since R is reflexive relation on A, therefore $(a, a) \in R \forall a \in A$.

Then the minimum no. of ordered pairs in R is 10.

Therefore $m \geq 10$

43. If $|x-6| = |x^2 - 4x| - |x^2 - 5x + 6|$, where x is a real variable

(a) x = (2, 5) (b) $x \in [2,3] \cup [6, \infty)$ (c) R – [2, 6] (d) None of these

Sol. **(b)**

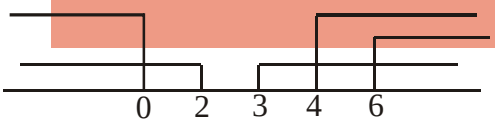
$|x-6| = |x^2 - 4| + |(x-3)(x-2)|$

Its one solution is

$x(x-4) \geq 0$ & $x^2 - 5x + 6 \geq 0$

And $x \geq 6$

$x \leq 0$ or $x \geq 4$ $x \leq 2$ & $x \geq 3$

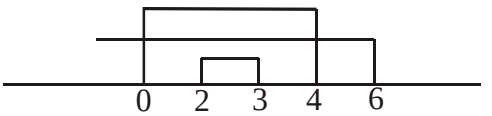


Its intersection $x \geq 6 \Rightarrow x \in [6, \infty)$

Its second solution $x(x-4) \leq 0$ & $x^2 - 5x + 6 \leq 0$

$0 \leq x \leq 4$ & $2 \leq x \leq 3$ & $x < 6$

And $x \leq 6$



Its intersection is $2 \le x \le 3 \Rightarrow x \in [2,3]$

Union of both solutions is $x \in [2,3] \cup [6,\infty)$

44. The range of values of θ in the interval $(0,\pi)$ such that the points (3, 2) and $(\cos\theta, \sin\theta)$ lie on the same sides of the line x + y – 1 = 0, is

(a) $\left(0,\frac{3\pi}{4}\right)$ (b) $\left(0,\frac{\pi}{2}\right)$ (c) $\left(0,\frac{\pi}{3}\right)$ (d) $\left(0,\frac{\pi}{4}\right)$

Sol. **(b)**

$L: x+y-1=0$

As $(3,2)$ & $(\cos\theta, \sin\theta)$ lies on same side of line

$L:(3,2)3+2-1=4>0$

So, $L:(\cos\theta,\sin\theta) \equiv \sin\theta+\cos\theta-1>0$

$\Rightarrow \sqrt{2}\left[\frac{1}{\sqrt{2}}\cos\theta+\frac{1}{\sqrt{2}}\sin\theta\right]>1$

$\sin\left(\theta+\frac{\pi}{4}\right)>\frac{1}{\sqrt{2}}$

$\Rightarrow \frac{\pi}{4}<\theta+\frac{\pi}{4}<\frac{3\pi}{4}$

$\Rightarrow 0<\theta<\frac{\pi}{2}$

45. Which of the following number is the coefficient of x^{100} in the expansion of $\log_e\left(\frac{1+x}{1+x^2}\right), |x|<1$?

(a) 0.01 (b) 0.02 (c) –0.03 (d) –0.01

Sol. **(a)**

$\log_e(1+x) = x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}-.....-\frac{x^{100}}{100}+...$

$\ln(1+x^2) = x^2-\frac{x^4}{2}+\frac{x^6}{3}-\frac{x^8}{4}+.....-\frac{(x^2)^{50}}{50}$

Coefficient of x^{100} in $\ln\left(\frac{1+x}{1+x^2}\right)$

Coefficient of x^{100} in $\ln(1+x)-\ln(1+x^2)$

$=-\frac{1}{100}+\frac{1}{50}=0.01$

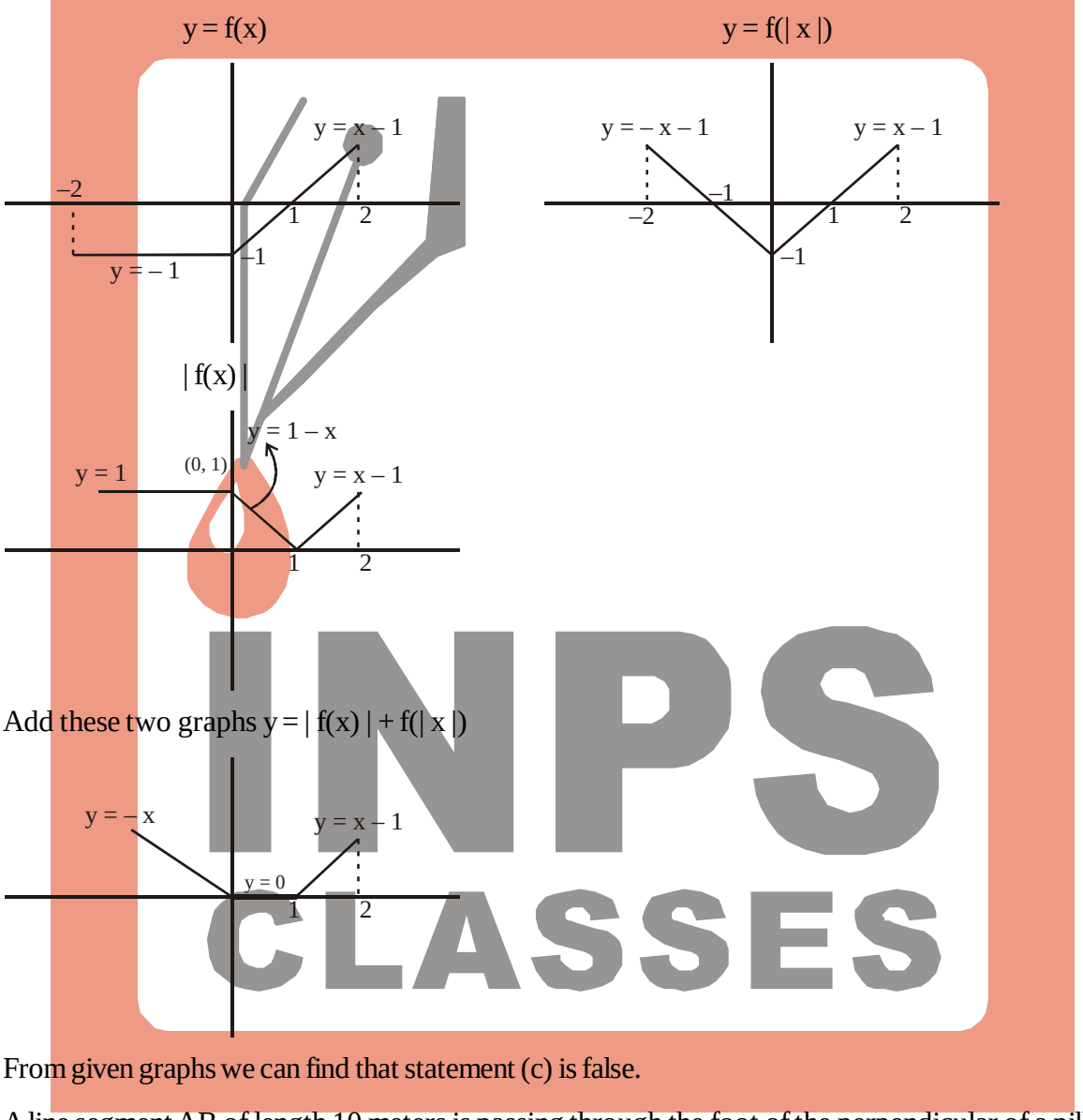
46. A real valued function f is defined as $f(x)=\begin{cases}-1 & -2\le x\le 0\\ x-1 & 0\le x\le 2\end{cases}$. Which of the following statement is FALSE?

(a) $f(|x|)=|x|-1,$ if $0\le x\le 1$ (b) $|f(x)|=x-1$, if $1\le x\le 2$

(c) $f(|x|)+|f(x)|=1,$ if $0\le x\le 1$ (d) $f(|x|)-|f(x)|=0,$ if $1\le x\le 2$

Sol. **(c)**

Graphical solution :



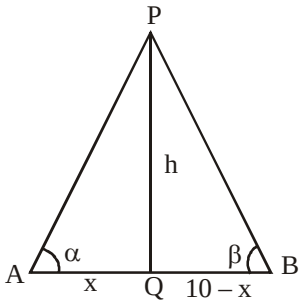
Add these two graphs $y=|f(x)|+f(|x|)$

From given graphs we can find that statement (c) is false.

47. A line segment AB of length 10 meters is passing through the foot of the perpendicular of a pillar, which is standing at right angle to the ground. Tip of the pillar subtends angles $\tan^{-1}3$ and $\tan^{-1}2$ at A and B respectively. Which of the following choice represents the height of the pillar?

(a) 10 meter (b) 8 meter (c) 12 meter (d) 15 meter

Sol. **(c)**



Let $\alpha = \tan^{-1} 3 \Rightarrow \tan\alpha = 3$ and $\beta = \tan^{-1} 2 \Rightarrow \tan\beta = 2$

In ΔAPQ

$$\tan\alpha = \frac{h}{x} = 3 \Rightarrow x = \frac{h}{3}$$

In ΔBPQ

$$\tan\beta = \frac{h}{10-x} = 2 \Rightarrow h = 20 - 2x$$

$$\Rightarrow h = 20 - 2h/3$$

$$\Rightarrow h = 12$$

48. If a vector having magnitude of 5 units, makes equal angle with each of the three mutually perpendicular axes, then the sum of the magnitude of the projections on each of the axis is

(a) 15/3 unit (b) $5\sqrt{3}$ unit (c) $\frac{15\sqrt{3}}{2}$ units (d) None of these

Sol. **(b)**

$|\vec{v}| = 5$

As $\vec{v}$ makes equal angle with all the three axis

$$\vec{v} = x\hat{i} + x\hat{j} + x\hat{k}$$

$$|\vec{v}| = \sqrt{3}x = 5$$

$$x = \frac{5}{\sqrt{3}}$$

Sum of component on x, y, z axis

$$x + x + x = 3\left(\frac{5}{\sqrt{3}}\right) = 5\sqrt{3}.$$

49. Bag I contains 3 red, 4 black and 3 white balls and Bag II contains 2 red, 5 black and 2 white balls. One balls is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be black in colour. Then the probability, that the transferred is red, is:

(a) 4/9 (b) 5/18 (c) 1/6 (d) 3/10

Sol. **(b)**

3R	2R
4B	5B
3W	2W

A : Drawn ball from boy II is black
B : Red ball transferred

$$P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}$$

$$=\frac{\frac{3}{9}\times\frac{5}{10}}{\frac{3}{9}\times\frac{5}{10}+\frac{4}{9}\times\frac{6}{10}+\frac{3}{9}\times\frac{5}{100}}=\frac{15}{15+24+15}=\frac{15}{54}=\frac{5}{18}$$

50. Let $f(x)=\frac{(x^2-1)}{(|x|-1)}$. Then the value of $\lim_{x\to -1} f(x)$ is

(a) –1 (b) 1 (c) 2 (c) 3

Sol. **(c)**

$$\lim_{x\to -1}\frac{x^2-1}{-(x+1)}=\frac{(x-1)(x+1)}{-(x+1)}=2$$

Reasoning

51. Complete the series: 3, 10, 24, 45, 73, ...

(a) 69 (b) 91 (c) 108 (d) 121

Ans. **(c)**

52. Pointing towards a person in the photograph, Anjali said, "He is the only son of the father of my sister's brother". How is that person related to Anjali?

(a) father (b) mother (c) cousing (d) None of these

Ans. **(d)**

53. Book : Publisher : : Film : ?

(a) Director (b) Producer (c) Editor (d) Writer

Ans. **(b)**

54. A sum of money distributed among four person P, Q, R, S in ratio 2 : 5 : 4 : 3. If Q get Rs. 2000 more than S, then what will be the total amount

(a) 18000 (b) 16000 (c) 14000 (d) 15000

Ans. **(c)**

55. P, Q, R, S, T, U, V, W are sitting around a table in the same order, for group discussion at equal distances. Their position are clockwise. If V sit in north, then what will be the position of 'S'

(a) East (b) South (c) South – East (d) South – West

Ans. **(d)**

56. If yellow is called white, white is called black, black is called green, green is called pink, pink is called blue and blue is called water, what is the colour of sky.

(a) Black (b) Water (c) White (d) Blue

Ans. **(b)**

57. Which of the following can be formed from "RECCOMMENDATION" word letters

(a) MEDIATE (b) COMMUNICATE (c) MEDICINE (d) REMAINDER

Ans. **(a)**

58. How many meaningful words can be formed with the first, the third, the seventh and the ninth letters of the word SEPERATION using each letter only once in each word?

(a) 3 (b) 4 (c) more than 4 (d) 2

Ans. **(c)**

59. Find a number from the given options which best completes the series: 39, 416, 525, ..., 749, 864

(a) 439 (b) 436 (c) 636 (d) 644

Ans. **(c)**

60. A can do a piece of work in 10 days and B can do a work in 20 days after 4 days B left and C joins then A and C to work together in 2 days. In how many days C alone do the work

(a) 12 days (b) 15 days (c) 10 days (d) 20 days

Ans. **(c)**

61. Thirty-six vehicles are parked in a parking lot in a single row. After the first car, there is one scooter. After the second car, there are two scooters. After the third car, there are three scooters and so on. Work out the number of scooter in the second half row.

(a) 15 (b) 17 (c) 12 (d) 10

Ans. **(a)**

62. Deepa moved a distance of 75 metre towards the north. She then turned left and walking for about 25 meter, turned left again and waked 80 meters. Finally, she turned to the right at an angle of 45°. In which direction was she moving finally?

(a) North – East (b) North – West (c) South – East (d) South – West

Ans. **(d)**

63. A 30 litres mixture of milk and water 10% water. How much milk should be added so that the percentage of water in the mixture in the mixture comes down to 2%?

(a) 120 (b) 80 (c) 90 (d) 60

Ans. **(a)**

64. Rakesh says to Mahesh – "I am as old as you were when I was one-third as you are". If the sum of their ages is 60 year, find the present age of the Mahesh (in years).

(a) 45 (b) 36 (c) 30 (d) 24

Ans. **(b)**

65. Hari invested an amount at a certain rate of interest on simple interest and he got 60% more amount after 8 years. If he invest Rs. 9600 at the same rate of interest on simple interest then find total interest he would get after four years.

(a) Rs. 2520 (b) Rs. 2260 (c) Rs. 2880 (d) Rs. 2160

Ans. **(c)**

66. Ritu purchased 20 dozen of Banana at the rate of Rs. 375 per dozen. She sold each one of them at the rate of Rs. 33. What was the percentage profit.

(a) 12.3% (b) 5.6% (c) 6.5% (d) 10%

Ans. **(b)**

67. A university is offering elective courses in Mathematics, Economics and Sociology. Each of its 100 undergraduate students has to opt for at least one of these electives. Course enrollment data showed that 47 students enrolled for Mathematics, 47 students enrolled for Economics and 57 students enrolled for Sociology. If 7 students enrolled for all three courses, how many students enrolled for exactly one course?

(a) 60 (b) 56 (c) 58 (d) Cann't say

Ans. **(b)**

68. Yamini has atleast two sister younger than her, Rakhi, Pooja, Yamini and Sweta are four sister. Pooja is younger than Rakhi who is not oldest. Sweta is older than Pooja but younger than rakhi, then who is oldest among them?

(a) Sweta (b) Yamini (c) Pooja (d) Rakhi

Ans. **(b)**

69. **Statements:**
Statement I: At most teachers are boys.
Statement II: Some boys are students.
Conclusion:
I. Some teacher are students.
II. Some students are boys.
(a) Neither I or II follows (b) Neither I nor II follows (c) Only II follows (d) Only I follows

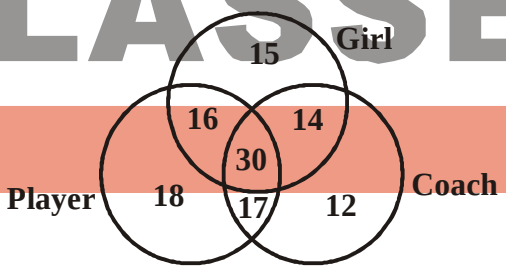
Ans. **(d)**

70. Identify the next two numbers in the following sequence: 17, 20, 9, 12, 5, 6, 3, 2, ?, ?

(a) (0, 2) (b) (1, 2) (c) (2, 0) (d) (2, 1)

Ans. **(c)**

71. Study the diagram and answer the question that follows:



(a) 16 (b) 31 (c) 15 (d) 61

Ans. **(a)**

72. Three persons A, B and C are standing in queue. There are five persons between A and B and eight persons between B and C. If there are three persons ahead of C and 21 behind A, then what could be the minimum number of persons in the queue?

(a) 40 (b) 27 (c) 28 (d) 41

Ans. **(c)**

73. Hemant deposits 10% of his salary in PF. He saves 30% of remaining salary. The ratio of his expenses on medicine and groceries is 3 : 4 of remaining salary after saving. If his expenses in medicine was Rs. 2700, then find his monthly salary.

(a) Rs. 30,000 (b) Rs. 20,000 (c) Rs. 10,000 (d) Rs. 15,000

Ans. (c)

74. Rajdhani Train running at a speed of 54 km/hr crosses a platform of length same as that of the train in 36 sec. If a Duranto train, which is 230 meters long crosses the same platform in 25 sec, then find speed of Duranto train (in km/hr)?

(a) 54 km/h (b) 72 km/h (c) 84 km/h (d) 90 km/hr

Ans. (b)

75. If $20 - 10$ means 200, $8 \div 4$ mean 12, 6×2 means 4 then $\left[(100-10)\times(1000\div1000)\right]+(100\times10) =$

(a) 20 (b) 0 (c) 1090 (d) None of these

Ans. (b)

76. If DENMARK in coded on FCPKCPM Then code SINGAPORE of which option.

(a) UGPECNQPG (b) UGPFCNRPG (c) UGPEDNQTC (d) UGPECNQTC

Ans. (a)

77. In the given word "LAVISHLY" if all the consonants replaced with its previous letter and all the vowels replaced with its next letter after that remove all the repeated letter and arranged them in alphabetical order then, which of the following letters is 3rd from the left end

(a) R (b) U (c) J (d) G

Ans. (c)

78. In the half yearly exam only 60% of the student were passes. Out of these (passed in half-yearly) only 70% students are passed in annual exam, out of remaining students (who fail in half-yearly exam) 80% passed in annual exam. What percent of the students passed the annual exam?

(a) 76% (b) 72% (c) 74% (d) 65%

Ans. (c)

79. Arrange in correct order

(1) Database (2) Analysis (3) Survery (4) Policy formation (5) Interpretation

(a) 2, 1, 5, 3, 4 (b) 5, 4, 3, 1, 2 (c) 3, 1, 2, 4, 5 (d) 3, 1, 2, 5, 4

Ans. (d)

80. Two friends A and B were standing at the diagonally opposite corners of a rectangular plot whose perimeter is 100m. A first walked x meters along the length of the plot towards East and then y meters towards the South. B walked x meters along the breadth towards North and then y meters towards West. At the end of their walks, A and B were standing at the diagonally opposite corners of a smaller rectangular plot whose perimeter is 40m. How much distance did A walk?

(a) 15 (b) 40 (c) 25 (d) 50

Ans. (a)

81. On Monday, Akash run 4 km less than the distance he ran on Tuesday. Sanjay, who ran the same distance on Monday and Tuesday, ran 5 km more on Tuesday than the distance Akash ran on Monday. Find the difference between the distance covered by Akash and Sanjay over the two days.

(a) 5 km (b) 6 km (c) 4 km (d) 9 km

Ans. (b)

82. **Statement:**
No table is chair.
Not a single chair is stand.
Ever stand is statue.
Conclusion:
1. Some statue which are stand are table as well.
2. Some statue are not chair.

(a) If neither conclusion 1 nor 2 follow (b) If only conclusion 1 follows
(c) Only conclusion 2 follows (d) None of these

Ans. (c)

83. If the word IMPACT is coded as RNKZXG, then which of the following represents the code for the word DESCEND?

(a) WVHXVMW (b) MNBLNWM (c) MFBDFOM (d) MFBDNOM

Ans. (a)

84. A junior school is offering five after-school activities - Karate, Handwork, Music Dance and Gymnastics. Each of the five students – Leena, Megan, Neha, Omar and Pixia has subscrived to at least one activity. As per the school rules, anyone who subscribes to Gymnastics must also subscribe to Dance. Karate and Handwork must always be subscribed together. Music and Dance cannot be subscribed together.
The following information is available about the student's subscriptions.
⇒ Megan subscribed to four activities.
⇒ Leena subscribed to Gymnastics but not Karate.
⇒ Pixie subscribed to only one activity and is the only one subscribe to that activity.
⇒ Omar subscribed to three activities.
⇒ Neha subscribed to only one activity.
How many activities are subscribed by exactly two people each?
(a) 1 (b) 4 (c) 2 (d) 3

Ans. (d)

85. If 'E' stands for +, 'F' stands for '–', 'M' stands for '×', 'N' stands for '÷', then 19 M 5 E 39 N 3 F 8 = ?

(a) 105 (b) 100 (c) 95 (d) 90

Ans. (b)

86. In a reality show, two judges independently provided marks based on the performance of the participants. If the marks provided by the second judge are given by y = 1 + x, where x is the marks provided by the first judge. Then for a participant
(a) Ranks given by both the judge differ by 2
(b) Rank given by the second judge is more than that of the first judge
(c) Ranks given by both the judges are same.
(d) Rank given by the first judge is more than that of the second judge

Ans. (c)

87. Fill the blanks with the most appropriate combination of options.

Further, to augment bond market liquidity, corporates need to be encouraged to _____ exiting bonds under the same International Securities Identification Number, to duly shore up floating _____.

(a) affect, negotiate (b) abandon, imply (c) precaution, abstract (d) reissue, stocks

Ans. (d)

88. A vessel contains total 95 litre mixture of milk & water in the ratio of 15 : 4 respectively P litre of mixture taken out from the vessel and 18 litres water added in the remaining mixture, then the new ratio of milk to water becomes 3 : 2, find the value of P?

(a) 57 (b) 19 (c) 27.5 (d) 38

Ans. (d)

89. For security reasons, a bank manager decided to encrypt the account number in the server. If A/C No. 46873 is coded as 91317157, then 52191 is coded as

(a) 4108041 (b) 5219152 (c) 1153193 (d) 1043293

Ans. (c)

90. If 3 is subtracted from the middle digit of each of the following numbers and then the position of the digits are reversed, which of the following will be last digit of the middle number after they are arranged in descending order?

589 362 554 371 442

(a) 3 (b) 2 (c) 4 (d) 1

Ans. (a)

Computer

91. The maximum and minimum value represented in signed 16 bit 2's complement representations are

(a) –16684 and 16383 (b) 0 and 32767 (c) 0 and 65535 (d) –32678 and 32767

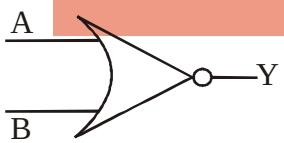
Ans. (d)

92. The time required for fetching & execution one machine instruction is

(a) Delay time (b) CPU cycle (c) Real time (d) Seek time

Ans. (b)

93. Consider the circuit shown below and find minimum number of NAND gates required to design it.



(a) 4 (b) 6 (c) 3 (d) 5

Ans. (a)

94. If a processor clock is rated as 2500 million cycles per seconds, then its clock period is

(a) 2.50×10^{-10}sec (b) 4.0×10^{-10}sec (c) 1.0×10^{-10}sec (d) 5.0×10^{-10}sec

Ans. (b)

95. Which of the following registers is used to keep track of address of memory location where the next instruction is located?

(a) Program counter (b) Memory Address Register

(c) Memory data register (d) Instruction counters

Ans. **(a)**

96. How many 32K × 1 RAM chips are needed to provided a memory capacity of 256 K bytes?

(a) 8 (b) 128 (c) 64 (d) 32

Ans. **(c)**

97. The number of minterms in a **n** variable truth table is

(a) n^2 (b) $(n-1)^2$ (c) 2^n (d) 2^{n-1}

Ans. **(c)**

98. A bulb in the staircase has two switches, one switch is at the ground floor and the other one is at the first floor. The bulb can be turned ON and also can be turned OFF by any of the switches irrespective of the state of the other switch. The logic of the switching of the bulb resembles.

(a) XOR Gate (b) AND Gate (c) OR Gate (d) XNOR Gate

Ans. **(a)**

99. What is a potential problem of 1's complement representation of numbers?

(a) Binary substructions are not possible (b) There are two different representations of zero

(c) Multiplication of two numbers cannot be carried out (d) Binary additions are not possible.

Ans. **(b)**

100. A wrong sentence related to FAT 32 and NTFS file system is

(a) FAT 32 has lower disk utilisation compared to NTFS

(b) Read and Write speeds of NTFS are faster than that of FAT 32

(c) FAT 32 store individual files of size up to 32 GB

(d) NTFS stands for New Technology File System

Ans. **(c)**

101. Consider the following minterm expression for F : $F(P, Q, R, S) = \sum 0, 2, 5, 7, 8, 10, 13, 15$. The minters 2, 7, 8 and 13 are don't care terms. The minimal sum of products form for F is

(a) $\overline{Q}S + Q\overline{S}$ (b) $\overline{Q}\,\overline{S} + QS$

(c) $\overline{Q}R\overline{S} + \overline{Q}RS + Q\overline{R}S + QRS$ (d) $\overline{P}\,\overline{Q}\,\overline{S} + \overline{P}QS + PQS + P\overline{Q}\,\overline{S}$

Ans. **(b)**

102. The reduced form of the Boolean function $F = xyz + xyz' + x'yz + zy'z$ is

(a) xy + yz (b) x + yz + xz (c) x + y + z (d) xy + yz + xz

Ans. **(d)**

103. Suppose we have a 10-bit computer that uses 10-bit into (2's complement representation). The number representation of –35 is

(a) 0000100011 (b) 1100100011 (c) 1111011101 (d) 1111011101

Ans. (c)

104. Consider the following Boolean expression for F: F(P, Q, R, S) = $PQ + \overline{P}QR + \overline{P}Q\overline{R}S$. The minimum sum of products form of F is

(a) PQ + QR + QS (b) P + Q + R + S (c) $\overline{P} + \overline{Q} + \overline{R} + \overline{S}$ (d) $\overline{P}R + \overline{P}\overline{R}S + P$

Ans. (a)

105. What is the name of the storage device that compensates the difference in rates of flow of data from one device to another?

(a) Cache (b) Buffer (c) Concentrator (d) RAM

Ans. (b)

106. Equavalent of the decimal number $(25.375)_{10}$ in binary form

(a) $(11001.011)_{10}$ (b) $(11101.011)_{10}$ (c) $(11011.111)_{10}$ (d) $(11001.101)_{10}$

Ans. (a)

107. A CPU generates 32-bit virtual addresses. The page size is 4 KB. The processor has a translation look-aside buffer (TLB) which can hold a total of 128 page table entries and is 4-way set associative. The minimum size of the TLB tag is:

(a) 13 bits (b) 20 bits (c) 11 bits (d) 15 bits

Ans. (d)

108. Which of the following is true about Von Newmann architecture?
(a) It has separate storage for input/output operations
(b) It has a separate processing unit for data and instructions
(c) It has separate memory for data and instructions
(d) It has a single memory unit for both data and instructions

Ans. (d)

109. Let $\oplus$ and $\odot$ denote the Exclusive – OR and Exclusive – NOR operations respectively. Which one of the following is not correct?

(a) $\overline{P} \oplus \overline{Q} = P \odot Q$ (b) $\overline{P} \oplus Q = P \odot Q$ (c) $\overline{P} \oplus \overline{Q} = P \oplus Q$ (d) $(P \oplus \overline{P}) \oplus Q = (P \odot \overline{P}) \odot \overline{Q}$

Ans. (d)

110. Suppose we have a 10-bit computer that uses 10-bit floating point computational unit (Flot number uses IEEE floating-point arithmetic where a floating point number has 1 sign bit, 5 exponent bits, and 4 fraction bits). The representation for $+\infty$ (plus infinity) is

(a) 0 11111 0000 (b) 1 11111 0000 (c) 0 00000 1111 (d) 0 11111 1111

Ans. (a)

English

111. Comprehension:

Science and religion – the two terms have come to signify a mutual antagonism. The two, it is commonly declared, are poles apart; their spheres of activity and their methods differ widely, so much so that they are considered to be irreconcilable.

On the face of it, science and religion appear to be the two opposite poles of man's consciousness. Science is basically concerned with the material world; its efforts are directed towards unraveling the "how" of reality while religion is concerned with the "why" of reality. Science deals with analyzing tangible entities into its minutest parts, and then arrives at conclusions about the way in which tangible realities are organized. While science is analytical, religion takes the ultimate reality for granted. Religion follows the metaphysical path; the concept of God is ultimately a matter of faith and it is this faith which is the basis of the religious man's attribution of a design or meaning for the reality.

The modes of action are different in science and religion. Science relies on experiment, whereas religion is based on experience. Any religious experience, whether it is Christ's or Ramakrishna's, is personal and subjective. Science, on the other hand, is marked by objectivity. Theory has to be corroborated by tangible proof. Science benefits mankind by providing material comforts. The frontiers of science do not end in knowledge but are extended to the formation of appliances for actual use. Science, it has been somewhat unfairly charged, cultivates the materialistic thinking. However, it has to be admitted that the mental attitude promoted by religion is entirely different, while the basis of scientific progress is unbridled curiosity and courageous endeavour, the truly religious spirit cavils at such presumption that man's mind can penetrate the mysteries of the universe. Science promotes fearless inquiry while an essential ingredient of religion is the humility born of fear of God. Science incorporates a love of experimental knowledge, while religion does not believe in the rational approach.

Which of the following statements according to the passage is correct:

(a) The religious spirit assumes that human mind can penetrate the mysteries of the universe.

(b) Science follows the metaphysical path.

(c) Science believes in the humility born of fear of God.

(d) Religion believes in ultimate reality

Ans. (d)

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Q. Which of the following reasons according to the passage provide material comforts to people in case of science?

(a) Trangible proofs of the theories of science (b) Materialistic thinking being cultivated by science

(c) Promotion of fearless inquiry by science (d) The subjectivity of science

Ans. **(a)**

113. Select the most appropriate meaning of the underlined idiom in the given sentence:

Off and on, I take a break from my hectic schedule to refresh myself.

(a) Periodically (b) Rarely (c) Seldom (d) Immediately

Ans. **(a)**

114. I have _____ umbrella. I bought it _____ year ago.

(a) A, An (b) An, A (c) An, The (d) Then, An

Ans. **(b)**

115. Synonym for "Nonplussed" is

(a) Flummoxed (b) Dumbfounded (c) Befuddled (d) Oriented

Ans. **(d)**

116. Select the most appropriate preposition to fill in the blank.

A baby sister is someone who look _____ other people's children.

(a) after (b) for (c) on (d) over

Ans. **(a)**

117. Select the most appropriate preposition to fill in the blank.

We haven't been to Delhi _____ almost five years.

(a) to (b) since (c) from (d) for

Ans. **(d)**

118. Meaning of "Abrogate" is

(a) Abolish (b) Absorb (c) Abstract (d) Ablaze

Ans. **(a)**

119. Choose the best option that indicates the change of voice for the sentence given below:

They sent for a doctor because Pamela had fainted

(a) Pamela fainted and a doctor was sent for

(b) A doctor was sent for them because Pamela has fainted

(c) A doctor was sent for because Pamela had fainted

(d) Pamela had sent for a doctor because they had fainted

Ans. **(c)**

120. Antonym for "Spendthrift" is

(a) Profligate (b) Extravagant (c) Frugal (d) Squanderer

Ans. **(c)**