INPS CLASSES

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Master of Computer Application (MCA) Entrance Examination

Solved Paper 2016

Time Allowed : 180 min **Max. Marks : 100**

Instructions for Candidates

Read the following instructions carefully.

1. All questions in this solved paper are of objective type.
2. There is a total of 100 questions.
3. (Q. Nos. 1-50) consist of Mathematics.
4. (Q. Nos. 51-60) consist of English Language and Comprehension.
5. (Q. Nos. 61-80) consist of Computer Awareness.
6. (Q. Nos. 81-100) consist of Logical and Analytical Ability.

MATHEMATICS

1. The value of the expression

$$2\left(1+\frac{1}{\omega}\right)\left(1+\frac{1}{\omega^2}\right)+3\left(2+\frac{1}{\omega}\right)\left(2+\frac{1}{\omega^2}\right)+\ldots+(n+1)\left(n+\frac{1}{\omega}\right)\left(n+\frac{1}{\omega^2}\right) \text{ is}$$

(a) $\left[\frac{n(n+1)}{2}\right]^2$ (b) $\left[\frac{n(n+1)}{2}\right]^2 - n$

(c) $\left[\frac{n(n+1)}{2}\right]^2 + n$ (d) None of these

Sol. ***(c)*** $2\left(1+\frac{1}{\omega}\right)\left(1+\frac{1}{\omega^2}\right)+3\left(2+\frac{1}{\omega}\right)\left(2+\frac{1}{\omega^2}\right)+\ldots+(n+1)\left(n+\frac{1}{\omega}\right)\left(n+\frac{1}{\omega^2}\right)$

$= (1+\omega)(1+\omega^2)+3(2+\omega)(2+\omega^2)+\ldots+(n+1)(n+\omega)(n+\omega^2)$

$= \sum_{r=1}^{n}(r+1)(r+\omega)(r+\omega^2) = \sum_{r=1}^{n}(r+1)[r^2+(\omega+\omega^2)r+\omega^3]$

$= \sum_{r=1}^{n}(r+1)(r^2-r+1) = \sum_{r=1}^{n}(r^3+1) = \left[\frac{n(n+1)}{2}\right]^2 + n$

2. All chords of the curve $3x^2 - y^2 - 2x + 4y = 0$ which subtend a right angle at the origin pass through the fix point.

(a) (1, 2) (b) (1, – 2) (c) (– 1, 2) (d) (– 1, – 2)

Sol. ***(b)*** Given equation of the curve is

$$3x^2 - y^2 - 2x + 4y = 0 \quad \ldots(i)$$

Let the equation of one of the chord be

$$y = mx + c$$

$$\Rightarrow \quad \frac{y - mx}{c} = 1 \quad \ldots(ii)$$

On making Eq. (i) homogeneous, we get

$$3x^2 - y^2 + (-2x + 4y)\left(\frac{y - mx}{c}\right) = 0$$

$$\Rightarrow \quad x^2(3c + 2m) + y^2(-c + 4) - 2xy - 4mxy = 0$$

which represent a pair of straight lines passing through origin.

Since, the angle subtended is a right angle

$$\therefore \quad 3c + 2m - c + 4 = 0$$

$$c = -m + 2$$

Substituting the value of c in $y = mx + c$, we have

$$y = mx - m - 2$$

$$\Rightarrow \quad y + 2 = m(x - 1)$$

$\Rightarrow$ All such chord pass through a fixed point (1, – 2).

* Memory based

3. The equation $4x^2 - 24xy + 11y^2 = 0$ represents

(a) two parallel lines
(b) two perpendicular lines
(c) two lines through the origin
(d) a circle

Sol. *(c)* Here, $a = 4, b = 11$ and $h = -12$

$\because\ h^2 - ab = (-12)^2 - 4 \times 11 = 100$

$\therefore$ The two lines represented by given equation will be real and distinct which represent a pair of straight line passing through the origin.

4. The value of the integral $\int \frac{\log(x+1) - \log x}{x(x+1)}\,dx$ is

(a) $-\frac{1}{2}[\log(x+1)]^2 - \frac{1}{2}(\log x)^2 + \log(x+1)\log x + C$
(b) $-[\{\log(x+1)\}^2 - (\log x)^2] + \log(x+1)\log x + C$
(c) $\left[\log\left(1+\frac{1}{x}\right)\right]^2 + C$
(d) None of the above

Sol. *(a)* Let $I = \int \frac{\log(x+1) - \log x}{x(x+1)}\,dx$

$$= \int \frac{\log\left(1+\frac{1}{x}\right)}{x(x+1)}\,dx = \int \frac{\log\left(1+\frac{1}{x}\right)}{x^2\left(1+\frac{1}{x}\right)}\,dx$$

Put $1 + \frac{1}{x} = t$

$\Rightarrow \quad -\frac{1}{x^2}dx = dt$

$\therefore \quad I = -\int \frac{\log t}{t}\,dt = -\frac{1}{2}(\log t)^2 + C$

$$= -\frac{1}{2}\left[\log\left(1+\frac{1}{x}\right)\right]^2 + C$$

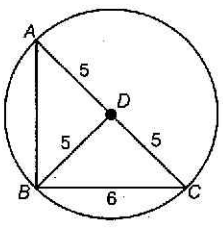
$$= -\frac{1}{2}[\log(x+1) - \log x]^2 + C$$

$$= -\frac{1}{2}\{\log(x+1)\}^2 - \frac{1}{2}(\log x)^2 + \log(x+1)\log x + C$$

5. ABC is a right angled triangle with $\angle B = 90°$, $a = 6$ cm. If the radius of the circumcircle is 5 cm. Then, the area of ΔABC is

(a) 25 cm^2
(b) 30 cm^2
(c) 36 cm^2
(d) 24 cm^2

Sol. *(d)* Let D be the centre of circumcircle $BD = 5$ cm



In ΔABC

$$AC^2 = AB^2 + BC^2$$

$\Rightarrow \quad 100 = AB^2 + 36$

$\Rightarrow \quad AB^2 = 64$

$\Rightarrow \quad AB = 8$

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times AB \times BC = \frac{1}{2} \times 8 \times 6 = 24\ \text{cm}^2$

6. The value of the determinant

$$\begin{vmatrix} 1 & \cos(\alpha-\beta) & \cos\alpha \\ \cos(\alpha-\beta) & 1 & \cos\beta \\ \cos\alpha & \cos\beta & 1 \end{vmatrix}$$ is

(a) 0 (b) 1
(c) $\alpha^2 - \beta^2$ (d) $\alpha^2 + \beta^2$

Sol. *(a)* Given, $\begin{vmatrix} 1 & \cos(\alpha-\beta) & \cos\alpha \\ \cos(\alpha-\beta) & 1 & \cos\beta \\ \cos\alpha & \cos\beta & 1 \end{vmatrix}$

is symmetric determinant

$\therefore$ Its value is

$1 + 2\cos(\alpha-\beta)\cos\alpha\cos\beta - \cos^2\alpha - \cos^2\beta - \cos^2(\alpha-\beta)$

$= 1 - \cos^2\alpha - \cos^2\beta - \cos(\alpha-\beta)$

$[\cos(\alpha-\beta) - 2\cos\alpha\cos\beta]$

$= 1 - \cos^2\alpha - \cos^2\beta - \cos(\alpha-\beta)$

$[\cos(\alpha-\beta) - \cos(\alpha+\beta) - \cos(\alpha-\beta)]$

$= 1 - \cos^2\alpha - \cos^2\beta + \cos(\alpha-\beta)\cos(\alpha+\beta)$

$= 1 - \cos^2\alpha - \cos^2\beta + \cos^2\alpha\cos^2\beta - \sin^2\alpha\sin^2\beta$

$= 1 - \cos^2\alpha - \cos^2\beta(1 - \cos^2\alpha) - \sin^2\alpha\sin^2\beta$

$= (1 - \cos^2\alpha)(1 - \cos^2\beta) - \sin^2\alpha\sin^2\beta$

$= \sin^2\alpha\sin^2\beta - \sin^2\alpha\sin^2\beta = 0$

7. The angle between the straight lines

$\mathbf{r} = (2-3t)\hat{\mathbf{i}} + (1+2t)\hat{\mathbf{j}} + (2+6t)\hat{\mathbf{k}}$ and

$\mathbf{r} = (1+4s)\hat{\mathbf{i}} + (2-s)\hat{\mathbf{j}} + (8s-1)\hat{\mathbf{k}}$ is

(a) $\cos^{-1}\left(\frac{\sqrt{41}}{34}\right)$ (b) $\cos^{-1}\left(\frac{21}{34}\right)$
(c) $\cos^{-1}\left(\frac{34}{63}\right)$ (d) $\cos^{-1}\left(\frac{5\sqrt{23}}{41}\right)$

Sol. *(c)* Given, lines can be rewritten as

$\mathbf{r} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}} + t(-3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 6\hat{\mathbf{k}})$ and

$\mathbf{r} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}} + s(4\hat{\mathbf{i}} - \hat{\mathbf{j}} + 8\hat{\mathbf{k}})$

Here, $a_1 = -3, b_1 = 2, c_1 = 6$

$a_2 = 4, b_2 = -1, c_2 = 8$

$\therefore \quad \cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

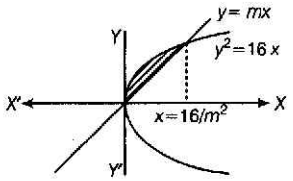
$\Rightarrow \quad \cos\theta = \frac{-3\times4 + 2\times(-1) + 6\times8}{\sqrt{9+4+36}\sqrt{16+1+64}} = \frac{34}{7\times9}$

$\Rightarrow \quad \theta = \cos^{-1}\left(\frac{34}{63}\right)$

8. Area bounded by the curve $y^2 = 16x$ and line $y = mx$ is $\frac{2}{3}$, then m is equal to

(a) 3 (b) 4 (c) 1 (d) 2

Sol. *(b)* Area $= \int_0^{16/m^2} (\sqrt{16x} - mx)\, dx = \frac{2}{3}$



$$\Rightarrow \left[4 \cdot \frac{2}{3} x^{3/2} - \frac{mx^2}{2}\right]_0^{16/m^2} = \frac{2}{3}$$

$$\Rightarrow \frac{1}{m^3}\left[\frac{512}{3} - \frac{256}{2}\right] = \frac{2}{3}$$

$$\Rightarrow m^3 = \frac{128}{3} \times \frac{3}{2} = 64$$

$$\therefore m = 4$$

9. The quadratic equation in x such that the arithmetic mean of its roots is 5 and geometric mean of the roots is 4, is given by

(a) $x^2 + 20x + 16 = 0$ (b) $x^2 + 10x + 16 = 0$
(c) $x^2 - 10x + 16 = 0$ (d) $x^2 - 10x - 16 = 0$

Sol. *(c)* Let α and β be the roots of given equation, then

$$\alpha + \beta = 10, \quad \alpha\beta = 16$$

∴Required equation is,

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\Rightarrow x^2 - 10x + 16 = 0$$

10. The solution set of the equation

$$\left[4\left(1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \ldots\right)\right]^{\log_2 x} = \left[54\left(1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots\right)\right]^{\log_x 2}$$ is

(a) $\left\{4, \frac{1}{4}\right\}$ (b) $\left\{2, \frac{1}{2}\right\}$ (c) $\{1, 2\}$ (d) $\left\{8, \frac{1}{8}\right\}$

Sol. *(a)* $\left[4\left(1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \ldots\right)\right]^{\log_2 x} = \left[54\left(1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots\right)\right]^{\log_x 2}$

$$\Rightarrow \left[4\left(\frac{1}{1 + 1/3}\right)\right]^{\log_2 x} = \left[54\left(\frac{1}{1 - 1/3}\right)\right]^{\log_x 2}$$

$$\Rightarrow \left[4\left(\frac{3}{4}\right)\right]^{\log_2 x} = \left[54 \times \frac{3}{2}\right]^{\log_x 2}$$

$$\Rightarrow 3^{\log_2 x} = 3^{4 \log_x 2}$$

$$\Rightarrow \log_2 x = 4 \log_x 2 = \frac{4}{\log_2 x}$$

$$\Rightarrow (\log_2 x)^2 = 4$$

$$\Rightarrow \log_2 x = \pm 2$$

If $\log_2 x = 2$

$$\Rightarrow x = 2^2 = 4 \text{ and if } \log_2 x = -2$$

$$\Rightarrow x = 2^{-2} = \frac{1}{4}$$

∴Solution set of the equation is $\left\{4, \frac{1}{4}\right\}$.

11. The tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ at

(a) (6, 7) (b) (– 6, 7) (c) (6, – 7) (d) (– 6, – 7)

Sol. *(d)* The tangent at (1, 7) to the parabola $x^2 = y - 6$ is

$x(1) = \frac{1}{2}(y + 7) - 6$ [replacing $x^2 \to xx_1$ and $2y \to y + y_1$]

$$\Rightarrow 2x = y + 7 - 12$$

$$\Rightarrow y = 2x + 5$$

which is also tangent to the circle

$$\Rightarrow x^2 + y^2 + 16x + 12y + c = 0$$

$$\Rightarrow x^2 + (2x + 5)^2 + 16x + 12(2x + 5) + c = 0$$

$$\Rightarrow 5x^2 + 60x + 85 + c = 0$$

must have equal roots $\alpha = \beta$ for above equation i.e.

$$\alpha + \beta = \frac{-60}{5}$$

on $\alpha = -6$ [as $\alpha = \beta$]

$$\therefore x = -6$$

and $y = 2x + 5 = -7$

Point of contact is (– 6, – 7).

12. $\frac{1}{1^3} \cdot \frac{2}{2} + \frac{2}{1^3 + 2^3} \cdot \frac{3}{2} + \frac{3}{1^3 + 2^3 + 3^3} \cdot \frac{4}{2} + \ldots + n$ terms equals

(a) $\left(\frac{n}{n+1}\right)^2$ (b) $\left(\frac{n}{n+1}\right)^3$
(c) $\left(\frac{n}{n+1}\right)$ (d) $\left(\frac{1}{n+1}\right)$

Sol. *(c)* $T_n = \frac{\frac{n(n+1)}{2 \cdot 2}}{1^3 + 2^3 + 3^3 + \ldots + n^3}$

$$= \frac{\frac{n(n+1)}{4}}{\left(\frac{n(n+1)}{2}\right)^2} = \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

$$\therefore T_n = \Sigma\left(\frac{1}{n} - \frac{1}{n+1}\right) = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \ldots + \left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$= 1 - \frac{1}{n+1} = \frac{n}{n+1}$$

13. The domain of the real valued function $f(x)=\sqrt{1-2x}+2\sin^{-1}\left(\frac{3x-1}{2}\right)$ is

(a) $\left[-\frac{1}{3},1\right]$ (b) $\left[\frac{1}{2},1\right]$ (c) $\left[-\frac{1}{2},\frac{1}{3}\right]$ (d) $\left[-\frac{1}{3},\frac{1}{2}\right]$

Sol. *(d)* The given function is

$$f(x)=\sqrt{1-2x}+2\sin^{-1}\left(\frac{3x-1}{2}\right)$$

For domain of $f(x), 1-2x\geq 0$ and $-1\leq\frac{3x-1}{2}\leq 1$

$\Rightarrow$ $x\leq\frac{1}{2}$ and $-2\leq 3x-1\leq 2$

$\Rightarrow$ $x\leq\frac{1}{2}$ and $-\frac{1}{3}\leq x\leq 1$

$\therefore$ Domain of $f(x)=\left[-\frac{1}{3},\frac{1}{2}\right]$

14. If the vector $\mathbf{a}+\lambda\mathbf{b}+3\mathbf{c}$, $-2\mathbf{a}+3\mathbf{b}-4\mathbf{c}$ and $\mathbf{a}-3\mathbf{b}+5\mathbf{c}$ are coplanar, then the value of λ is

(a) 2 (b) -1
(c) 1 (d) -2

Sol. *(d)* Since, given vectors are coplanar so it can be written as

$\mathbf{a}+\lambda\mathbf{b}+3\mathbf{c}=x(-2\mathbf{a}+3\mathbf{b}-4\mathbf{c})+y(\mathbf{a}-3\mathbf{b}+5\mathbf{c})$

On comparing the coefficient of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ on both sides, we get $-2x+y=1, 3x-3y=\lambda$ and $-4x+5y=3$

On solving above equations, we get

$$x=-\frac{1}{3}, y=\frac{1}{3}, \lambda=-2$$

15. The centroid of the triangle formed by the pair of straight line $12x^2-20xy+7y^2=0$ and the line $2x-3y+4=0$ is

(a) $\left(-\frac{7}{3},\frac{7}{3}\right)$ (b) $\left(-\frac{8}{3},\frac{8}{3}\right)$
(c) $\left(\frac{8}{3},\frac{8}{3}\right)$ (d) $\left(\frac{4}{3},\frac{4}{3}\right)$

Sol. *(c)* The separate equation of pair of straight lines of $12x^2-20xy+7y^2=0$ are $6x-7y=0$ and $2x-y=0$

Thus, equation of sides of triangle are

$$6x-7y=0 \quad ...(i)$$
$$2x-y=0 \quad ...(ii)$$

and $$2x-3y+4=0 \quad ...(iii)$$

On solving these equations, we get the vertices of a triangle $A(0, 0)$, $B(1, 2)$ and $C(7, 6)$

$\therefore$ Centroid of triangle is

$$\left(\frac{0+1+7}{3},\frac{0+2+6}{3}\right)=\left(\frac{8}{3},\frac{8}{3}\right)$$

16. If $f:R\to R$ is defined by $f(x)=x^2-6x-14$, then $f^{-1}(2)$ equal to

(a) $\{2, 8\}$ (b) $\{-2, 8\}$
(c) $\{-2, -8\}$ (d) $\{2, -8\}$

Sol. *(b)* Let $y=x^2-6x-14$

$\Rightarrow$ $y=(x-3)^2-23$

$\Rightarrow$ $x=\pm\sqrt{y+23}+3$

$\Rightarrow$ $f^{-1}(x)=\pm\sqrt{x+23}+3$

$\therefore$ $f^{-1}(2)=\pm\sqrt{25}+3=-2, 8$

17. If $2A+3B=\begin{bmatrix}2 & -1 & 4\\ 3 & 2 & 5\end{bmatrix}$ and $A+2B=\begin{bmatrix}5 & 0 & 3\\ 1 & 6 & 2\end{bmatrix}$, then B is

(a) $\begin{bmatrix}8 & -1 & 2\\ -1 & 10 & -1\end{bmatrix}$ (b) $\begin{bmatrix}8 & 1 & 2\\ -1 & 10 & -1\end{bmatrix}$
(c) $\begin{bmatrix}8 & 1 & -2\\ -1 & 10 & -1\end{bmatrix}$ (d) $\begin{bmatrix}8 & 1 & 2\\ 1 & 10 & 1\end{bmatrix}$

Sol. *(b)* We have,

$$2A+3B=\begin{bmatrix}2 & -1 & 4\\ 3 & 2 & 5\end{bmatrix} \quad ...(i)$$

and $$A+2B=\begin{bmatrix}5 & 0 & 3\\ 1 & 6 & 2\end{bmatrix} \quad ...(ii)$$

On solving Eqs. (i) and (ii), we get

$$B=2\begin{bmatrix}5 & 0 & 3\\ 1 & 6 & 2\end{bmatrix}-\begin{bmatrix}2 & -1 & 4\\ 3 & 2 & 5\end{bmatrix}=\begin{bmatrix}8 & 1 & 2\\ -1 & 10 & -1\end{bmatrix}$$

18. If in the expansion of $\left(3x-\frac{2}{x^2}\right)^{15}$, rth term is independent of x, then value of r is

(a) 6 (b) 10
(c) 9 (d) 12

Sol. *(a)* rth term in the expansion of $\left(3x-\frac{2}{x^2}\right)^{15}$ is

$$T_r={}^{15}C_{r-1}(3x)^{15-r+1}\left(\frac{-2}{x^2}\right)^{r-1}$$
$$={}^{15}C_{r-1}(3)^{15-r+1}(-2)^{r-1}(x)^{15-3r+3}$$

For the term independent of x, put

$$15-3r+3=0\Rightarrow r=6$$

19. The value of $\lim_{x\to 0}\left(\frac{\int_0^{x^2}\sec^2 t\,dt}{x\sin x}\right)$ is

(a) 3 (b) 2
(c) 1 (d) 0

Sol. *(c)* $\lim_{x\to 0}\left(\frac{\int_0^{x^2}\sec^2 t\,dt}{x\sin x}\right)=\lim_{x\to 0}\frac{\sec^2 x^2\,2x}{\sin x+x\cos x}$

$$=\lim_{x\to 0}\frac{2x\cdot\sec^2 x^2}{x\left(\frac{\sin x}{x}+\cos x\right)}$$

$$=\frac{2\times 1}{1+1}=1 \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

20. Form of the differential equation of all family of lines $y = mx + \frac{4}{m}$ by eliminating the arbitrary constant m is

(a) $\frac{d^2y}{dx^2} = 0$ (b) $x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} + 4 = 0$

(c) $x\left(\frac{dy}{dx}\right)^2 + y\left(\frac{dy}{dx}\right) + 4 = 0$ (d) $\frac{dy}{dx} = 0$

Sol. ***(b)*** $y = mx + \frac{4}{m}$ ...(i)

$\therefore$ $\frac{dy}{dx} = m$

From Eq. (i), we get $y = x\left(\frac{dy}{dx}\right) + \frac{4}{\left(\frac{dy}{dx}\right)}$

$\Rightarrow$ $y\left(\frac{dy}{dx}\right) = x\left(\frac{dy}{dx}\right)^2 + 4$

$\Rightarrow$ $x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} + 4 = 0$

which is the required differential equation.

21. A dictionary is printed consisting of 7 letters words only that can be made with a letters of the word CRICKET. If the words are printed at the alphabetical order as in an ordinary dictionary, then the number of words before the word CRICKET is

(a) 530 (b) 480 (c) 531 (d) 487

Sol. ***(a)*** Given word is CRICKET

Total number of letters are 7 out of which two letters 'C' are count as one

$\therefore$ Required number of ways of words before the word CRICKET

$= 5! \times 4 + 2 \times 4! + 2! = 480 + 48 + 2 = 530$

22. If a system of the equations

$(\alpha+1)^3 x + (\alpha+2)^3 y - (\alpha+3)^3 = 0$,

$(\alpha+1)x + (\alpha+2)y - (\alpha+3) = 0$ and $x + y - 1 = 0$ is consistent, then the value of α is

(a) 1 (b) 0 (c) -3 (d) -2

Sol. ***(d)*** Given equations are

$(\alpha+1)^3 x + (\alpha+2)^3 y - (\alpha+3)^3 = 0$

$(\alpha+1)x + (\alpha+2)y - (\alpha+3) = 0$

and $x + y - 1 = 0$

Since, this system of equation is consistent

$$\therefore \begin{vmatrix} (\alpha+1)^3 & (\alpha+2)^3 & -(\alpha+3)^3 \\ (\alpha+1) & (\alpha+2) & -(\alpha+3) \\ 1 & 1 & -1 \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 + C_1$, we get

$$\begin{vmatrix} (\alpha+1)^3 & (\alpha+2)^3 - (\alpha+1)^3 & (\alpha+1)^3 - (\alpha+3)^3 \\ (\alpha+1) & (\alpha+2) - (\alpha+1) & -(\alpha+3) + (\alpha+1) \\ 1 & 0 & 0 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} (\alpha+1)^3 & 3\alpha^2 + 9\alpha + 7 & -6\alpha^2 - 24\alpha - 26 \\ \alpha+1 & 1 & -2 \\ 1 & 0 & 0 \end{vmatrix} = 0$$

$\Rightarrow$ $-2(3\alpha^2 + 9\alpha + 7) + 6\alpha^2 + 24\alpha + 26 = 0$

$\Rightarrow$ $6\alpha + 12 = 0 \Rightarrow \alpha = -2$

23. If a, b and c be in arithmetic progression than the value of $(a+2b-c)(2b+c-a)(a+2b-c)$ is

(a) $16abc$ (b) $4abc$ (c) $8abc$ (d) $3abc$

Sol. ***(a)*** $2b = a + c$

Now, $(a + 2b - c)(2b + c - a)(a + 2b + c)$

$= (a + a + c - c)(a + c + c - a)(2b + 2b)$

$= 2a \cdot 2c \cdot 4b = 16abc$

24. $f(x) = x^3 - 6x^2 - 36x + 2$ is decreasing function, then $x \in$ is equal to

(a) $(6, \infty)$ (b) $(-\infty, -2)$

(c) $(-2, 6)$ (d) None of these

Sol. ***(c)*** Given, $f(x) = x^3 - 6x^2 - 36x + 2$

$f'(x) = 3x^2 - 12x - 36$

For decreasing $f'(x) < 0$

$\Rightarrow$ $3(x^2 - 4x - 12) < 0$

$\Rightarrow$ $(x-6)(x+2) < 0$

$\Rightarrow$ $-2 < x < 6$

$\Rightarrow$ $x \in (-2, 6)$

25. Sum of n terms of the series $\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{8} + \ldots$ is

(a) $2^n - 1$ (b) $2^{-n}(n-1)$

(c) $2^n(n-1) + 1$ (d) $2^{-n} + n - 1$

Sol. ***(a)*** Let $S = 1 + 3 + 7 + 15 + \ldots + T_n$

$\Rightarrow$ $\frac{S}{0} = \frac{1 + 3 + 7 + \ldots + T_{n-1} + T_n}{1 + 2 + 4 + 8 + \ldots - T_n}$

$\Rightarrow$ $T_n = 1 + 2 + 4 + \ldots n \text{ terms} = \frac{(2^n - 1)}{2 - 1} = 2^n - 1$

26. $\int_0^1 \log\left\{\sin\left(\frac{\pi x}{2}\right)\right\} dx$ is equal to

(a) $-\frac{\pi}{2}\log 2$ (b) $-\log 2$

(c) $\frac{-2}{\pi}\log 2$ (d) $\frac{\pi}{2}\log 2$

Sol. ***(b)*** Let $I = \int_0^1 \log\left\{\sin\left(\frac{\pi x}{2}\right)\right\} dx$

Put $\frac{\pi x}{2} = \theta$

$\Rightarrow$ $dx = \frac{2}{\pi} d\theta$

$\therefore$ $I = \frac{2}{\pi}\int_0^{\pi/2} \log\sin\theta \, d\theta$

$= \frac{2}{\pi}\left(-\frac{\pi}{2}\log 2\right) = -\log 2$

27. The average marks of boys in a class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is

(a) 40% (b) 20% (c) 80% (d) 60%

Sol. *(c)* Let the number of boys and girls be x and y.

Then, the total marks obtain by all boys and girls $= 52x + 42y$

$\therefore \quad 52x + 42y = 50(x + y)$

$\Rightarrow \quad 2x = 8y$

$\Rightarrow \quad x = 4y$

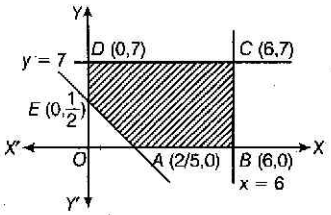
$\because$ Total number of students in the class $= x + y = 5y$

$\therefore$ Required percentage of boys $= \frac{4y}{5y} \times 100 = 80\%$

28. If given constraints are $5x - 14y \geq 2$, $x \leq 6$, $y \leq 7$, then the maximum value of the function

(a) 13 (b) 14 (c) 15 (d) 20

Sol. *(d)* Feasible region is $ABCDEA$ and $z = x + 2y$



At point $A\left(\frac{2}{5}, 0\right)$, $z = \frac{2}{5} + 0 = \frac{2}{5}$

At point $B(6, 0)$, $z = 6 + 0 = 6$

At point $C(6, 7)$, $z = 6 + 14 = 20$

At point $D(0, 7)$, $z = 0 + 2(7) = 14$

At point $E\left(0, \frac{1}{2}\right)$, $z = 0 + 2 \times \frac{1}{2} = 1$

So, the maximum value of z is 20.

29. In a ΔABC, $\cos\left(\frac{B + 2C + 3A}{2}\right) + \cos\left(\frac{A - B}{2}\right)$ is equal to

(a) -1 (b) 0 (c) 1 (d) 2

Sol. *(b)* In a ΔABC

$$A + B + C = \pi$$

$$\therefore \cos\left(\frac{B + 2C + 3A}{2}\right) + \cos\left(\frac{A - B}{2}\right)$$

$$= 2\cos\left(\frac{2C + 4A}{4}\right)\cos\left(\frac{2A + 2B + 2C}{4}\right)$$

$$= 2\cos\left(\frac{C + 2A}{2}\right)\cos\left(\frac{\pi}{2}\right) = 0$$

30. If $\mathbf{a} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ and

$\mathbf{b} = \hat{\mathbf{i}} \times (\mathbf{a} \times \hat{\mathbf{i}}) + \hat{\mathbf{j}} \times (\mathbf{a} \times \hat{\mathbf{j}}) + \hat{\mathbf{k}} \times (\mathbf{a} \times \hat{\mathbf{k}})$, then length of $\mathbf{b}$ is equal to

(a) $\sqrt{12}$ (b) $2\sqrt{12}$ (c) $3\sqrt{14}$ (d) $2\sqrt{14}$

Sol. *(d)* We have, $\mathbf{a} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

$\therefore \quad \mathbf{b} = \hat{\mathbf{i}} \times (\mathbf{a} \times \hat{\mathbf{i}}) + \hat{\mathbf{j}} \times (\mathbf{a} \times \hat{\mathbf{j}}) + \hat{\mathbf{k}} \times (\mathbf{a} \times \hat{\mathbf{k}})$ ...(i)

Now, $\hat{\mathbf{i}} \times (\mathbf{a} \times \hat{\mathbf{i}}) = (\hat{\mathbf{i}} \cdot \hat{\mathbf{i}})\,\mathbf{a} - (\hat{\mathbf{i}} \cdot \mathbf{a})\,\hat{\mathbf{i}}$

$= 1(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) - (1)\hat{\mathbf{k}} = 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

Similarly $\hat{\mathbf{j}} \times (\mathbf{a} \times \hat{\mathbf{j}}) = \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

and $\mathbf{k} \times (\mathbf{a} \times \hat{\mathbf{k}}) = \hat{\mathbf{i}} + 2\hat{\mathbf{j}}$

From Eq. (i), we get

$$\mathbf{b} = 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}} + \hat{\mathbf{i}} + 3\hat{\mathbf{k}} + \hat{\mathbf{i}} + 2\hat{\mathbf{j}} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}}$$

$\Rightarrow \quad |\mathbf{b}| = \sqrt{4 + 16 + 36} = 2\sqrt{14}$

31. If x satisfies the inequations $2x - 7 < 11$, $3x + 4 < -5$, then x lies in the interval

(a) $(-\infty, 3)$ (b) $(-\infty, 2)$ (c) $(-\infty, -3)$ (d) $(-\infty, \infty)$

Sol. *(c)* Given, $2x - 7 < 11, 3x + 4 < -5$

$\Rightarrow \quad x < 9,$

$x < -8$

$\Rightarrow \quad x < -3$

So, x lies in the interval $(-\infty, -3)$

32. In a class of 30 pupils 12 take needles works, 16 take Physics and 18 take History. If all the 30 students take atleast one subject and no one takes all three, then the number of pupils taking 2 subjects is

(a) 16 (b) 6

(c) 8 (d) 20

Sol. *(a)* Given, $n(N) = 12, n(P) = 16, n(H) = 18$

$$n(N \cup P \cup H) = 30$$

and $n(N \cap P \cap H) = 0$

Now, $n(N \cup P \cup H) = n(N) + n(P) + n(H)$

$$- n(N \cap P) - n(P \cap H) - n(H \cap N) + n(N \cap P \cap H)$$

$\Rightarrow \quad n(N \cap P) + n(P \cap H) + n(H \cap N)$

$$= (12 + 16 + 18) - 30 = 46 - 30 = 16$$

33. If $\sec^{-1}\sqrt{1 + x^2} + \text{cosec}^{-1}\frac{\sqrt{1 + y^2}}{y} + \cot^{-1}\frac{1}{z} = \pi$, then $x + y + z$ is equal to

(a) xyz (b) $2xyz$

(c) xyz^2 (d) x^2yz

Sol. *(a)* Given, $\sec^{-1}\sqrt{1 + x^2} + \text{cosec}^{-1}\frac{\sqrt{1 + y^2}}{y} + \cot^{-1}\frac{1}{z} = \pi$

$\therefore \quad \tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$

$\Rightarrow \quad \tan^{-1}\left(\frac{x + y + z - xyz}{1 - xy - yz - zx}\right) = \pi$

$\Rightarrow \quad x + y + z = xyz$

34. The general value of θ satisfying the equation $2\sin^2\theta - 3\sin\theta - 2 = 0$ is

(a) $n\pi + (-1)^{n+1}\frac{\pi}{6}$ (b) $n\pi + (-1)^n\frac{\pi}{2}$

(c) $n\pi + (-1)^n\frac{5\pi}{6}$ (d) $n\pi + (-1)^n\frac{7\pi}{6}$

Sol. *(a)* $2\sin^2\theta - 3\sin\theta - 2 = 0$

$\Rightarrow \quad (2\sin\theta + 1)(\sin\theta - 2) = 0$

$\Rightarrow \quad \sin\theta = -\frac{1}{2} \qquad [\because \sin\theta \neq 2]$

$\Rightarrow \quad \sin\theta = \sin\left(-\frac{\pi}{6}\right)$

$\Rightarrow \quad \theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$

$\Rightarrow \quad \theta = n\pi + (-1)^{n+1}\frac{\pi}{6}$

35. The distance between the planes $2x - 2y + z + 3 = 0$ and $4x - 4y + 2z + 5 = 0$ is

(a) 3 (b) 6 (c) $\frac{1}{6}$ (d) $\frac{1}{3}$

Sol. *(c)* Since, the planes $2x - 2y + z + 3 = 0$ and $2x - 2y + z + \frac{5}{2} = 0$ are parallel to each other,

$\therefore$ Distance between them

$$= \frac{|d_2 - d_1|}{\sqrt{a_1^2 + b_1^2 + c_1^2}} = \frac{\left|\frac{5}{2} - 3\right|}{\sqrt{4 + 4 + 1}} = \frac{1}{6}$$

36. For any complex number z, the minimum value of $|z| + |z - 1|$ is

(a) 0 (b) 1 (c) 2 (d) -1

Sol. *(b)* $\because \quad |-z| = |z|$ and $|z_1 + z_2| \leq |z_1| + |z_2|$

Now, $|z| + |z - 1| = |z| + |1 - z|$

$\geq |z + (1 - z)| = 1$

37. The solution of the differential equation $(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$ is

(a) $2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + C$ (b) $xe^{\tan^{-1} y} = \tan^{-1} y + C$

(c) $xe^{2\tan^{-1} y} = e^{\tan^{-1} y} + C$ (d) $(x - 2) = Ce^{-\tan^{-1} y}$

Sol. *(a)* Given, differential equation is

$$(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$$

$$\Rightarrow \quad (1 + y^2)\frac{dx}{dy} = -x + e^{\tan^{-1} y}$$

$$\Rightarrow \quad \frac{dx}{dy} + \frac{x}{1 + y^2} = \frac{e^{\tan^{-1} y}}{1 + y^2}$$

which is a linear differential equation

Here, $P = \frac{1}{1 + y^2}, Q = \frac{e^{\tan^{-1} y}}{1 + y^2}$

$$\Rightarrow \quad \text{IF} = e^{\int P\,dy} = e^{\int \frac{1}{1 + y^2} dy} = e^{\tan^{-1} y}$$

$\therefore$ Solution will be $x \cdot \text{IF} = \int Q \cdot \text{IF}\,dy + C$

$$\Rightarrow \quad xe^{\tan^{-1} y} = \int \frac{e^{\tan^{-1} y}}{1 + y^2} \cdot e^{\tan^{-1} y} + \frac{C}{2}$$

$$\Rightarrow \quad xe^{\tan^{-1} y} = \frac{e^{2\tan^{-1} y}}{2} + \frac{C}{2}$$

$$\therefore \quad 2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + C$$

38. The angle between the vectors $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$ when $\mathbf{a} = (1, 1, 4)$ and $\mathbf{b} = (1, -1, 4)$ is

(a) 45° (b) 90°

(c) 15° (d) 30°

Sol. *(b)* Here, $|\mathbf{a}| = \sqrt{1 + 1 + (4)^2} = 3\sqrt{2}$

and $|\mathbf{b}| = \sqrt{1 + (-1)^2 + (4)^2}$

$= 3\sqrt{2}$

$\therefore \quad |\mathbf{a}| = |\mathbf{b}|$

Now, $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}) = |\mathbf{a}|^2 - |\mathbf{b}|^2 = 0$

Hence, angle between them is 90°.

39. If $\sin^{-1}\left(\frac{x}{5}\right) + \text{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$, then the value of x is

(a) 1 (b) 3

(c) 4 (d) 5

Sol. *(b)* Given, $\sin^{-1}\left(\frac{x}{5}\right) + \text{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$

$$\Rightarrow \quad \sin^{-1}\left(\frac{x}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right) = \frac{\pi}{2}$$

$$\Rightarrow \quad \sin^{-1}\left(\frac{x}{5}\right) = \cos^{-1}\left(\frac{4}{5}\right)$$

$$\Rightarrow \quad \sin^{-1}\left(\frac{x}{5}\right) = \sin^{-1}\left(\frac{3}{5}\right)$$

$$\Rightarrow \quad x = 3$$

40. If $f : R \to R$ is defined by

$$f(x) = \begin{cases} \frac{2\sin x - \sin 2x}{2x\cos x}, & \text{if } x \neq 0 \\ a, & \text{if } x = 0 \end{cases}$$

then the value of a, so that f is continuous at 0, is

(a) 2 (b) 1

(c) -1 (d) 0

Sol. *(d)* Given,

$$f(x) = \begin{cases} \frac{2\sin x - \sin 2x}{2x\cos x}, & \text{if } x \neq 0 \\ a, & \text{if } x = 0 \end{cases}$$

Now, $\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{2\sin x - \sin 2x}{2x\cos x}$ $\left[\frac{0}{0} \text{ form}\right]$

$$= \lim_{x \to 0} \frac{2\cos x - 2\cos 2x}{2(\cos x - x\sin x)}$$

$$= \lim_{x \to 0} \frac{2 - 2}{2(1 - 0)} = 0$$

Since, $f(x)$ is continuous at $x = 0$

$$\therefore \quad f(0) = \lim_{x \to 0} f(x)$$

$$\Rightarrow \quad a = 0$$

41. Let R be the relation on the set R of all real numbers defined by aRb if $|a-b| \le 1$, then R is

(a) reflexive and symmetric (b) symmetric only
(c) transitive only (d) anti-symmetric only

Sol. *(a)* aRa if $|a-b| = 0 < 1$, which is true

$\therefore$ It is reflexive.

Now, aRb, $|a-b| \le 1$

$\Rightarrow$ $|b-a| \le 1 \Rightarrow aRb \Rightarrow bRa$

$\therefore$ It is symmetric.

42. The rate of change of the surface area of the sphere of radius r when the radius is increasing at the rate of 2 cm/s is proportional to

(a) $\frac{1}{r^2}$ (b) $\frac{1}{r}$ (c) r^2 (d) r

Sol. *(c)* Let surface area of sphere, $S = \frac{4}{3}\pi r^3$

$\Rightarrow$ $\frac{dS}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dS}{dt} = 4\pi r^2(2) = 8\pi r^2$

$\therefore$ $\frac{dS}{dt} \propto r^2$

43. $\int \frac{\sqrt{\tan x}}{\sin x \cdot \cos x}\,dx$ is equal to

(a) $2\sqrt{\cot x} + C$ (b) $\sqrt{\cot x} + C$
(c) $\sqrt{\tan x} + C$ (d) $2\sqrt{\tan x} + C$

Sol. *(d)* Divide numerator and denominator by $\cos^2 x$, we get

$$I = \int \frac{\sec^2 x\,dx}{\sqrt{\tan x}}$$

Put $\tan x = t$

$\Rightarrow$ $\sec^2 x\,dx = dt$

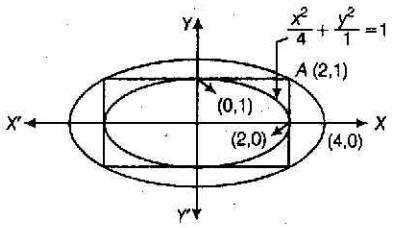
$\therefore$ $I = \int \frac{1}{\sqrt{t}}\,dt = 2\sqrt{t} = 2\sqrt{\tan x} + C$

44. The ellipse $x^2 + 4y^2 = 4$ is inscribed in a rectangle aligned with the coordinate axes, which is turn in inscribed in another ellipse that passes through the point (4, 0). Then, the equation of the ellipse is

(a) $x^2 + 12y^2 = 16$ (b) $4x^2 + 48y^2 = 48$
(c) $4x^2 + 64y^2 = 48$ (d) $x^2 + 16y^2 = 16$

Sol. *(a)* Let the equation of the required ellipse be $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$

But the ellipse passes through the point (2, 1)



$\Rightarrow$ $\frac{1}{4} + \frac{1}{b^2} = 1$

$\Rightarrow$ $\frac{1}{b^2} = \frac{3}{4}$

$\Rightarrow$ $b^2 = \frac{4}{3}$

Hence, equation is $\frac{x^2}{16} + \frac{3y^2}{4} = 1$

$\Rightarrow$ $x^2 + 12y^2 = 16$

45. $\int \frac{dx}{\cos x + \sqrt{3}\sin x}$ is equal to

(a) $\frac{1}{2}\log \tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$ (b) $\frac{1}{2}\log \tan\left(\frac{x}{2} - \frac{\pi}{12}\right) + C$
(c) $\log \tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$ (d) $\log \tan\left(\frac{x}{2} - \frac{\pi}{12}\right) + C$

Sol. *(a)* Let $I = \int \frac{dx}{2\left(\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right)}$

$= \frac{1}{2}\int \sec\left(x - \frac{\pi}{3}\right)dx = \frac{1}{2}\log \tan\left(\frac{x}{2} - \frac{\pi}{6} + \frac{\pi}{4}\right) + C$

$= \frac{1}{2}\log \tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$

46. Equation of the circle which of the circle $x^2 + y^2 - 2x = 0$ in the line $x + y = 2$

(a) $x^2 + y^2 - 2x + 4y + 3 = 0$
(b) $2(x^2 + y^2) + x + y + 1 = 0$
(c) $x^2 + y^2 - 4x - 2y + 4 = 0$
(d) None of the above

Sol. *(c)* Centre and radius of the given circle are (1, 0) and 1, respectively.

Let the centre of the image circle be (x_1, y_1).

Hence, (x_1, y_1) be the image of the point (1, 0) w.r.t. the line $x + y = 2$, then

$$\frac{x_1 - 1}{1} = \frac{y_1 - 0}{1} = \frac{-2[1(1) + 1(0) - 2]}{(1)^2 + (1)^2}$$

$\Rightarrow$ $\frac{x_1 - 1}{1} = \frac{y_1}{1} = 1$

$\Rightarrow$ $x_1 = 2,\ y_1 = 1$

$\therefore$ Equation of image circle is $(x-2)^2 + (y-1)^2 = 1^2$

$\Rightarrow$ $x^2 + y^2 - 4x - 2y + 4 = 0$

47. If $2\sec 2\alpha = \tan\beta + \cot\beta$, then one of the values of $\alpha + \beta$ is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$
(c) π (d) $n\pi - \frac{\pi}{4}, n \in I$

Sol. *(a)* Given, $2\sec 2\alpha = \tan\beta + \cot\beta$

$$\Rightarrow \quad 2\sec 2\alpha = \frac{1+\tan^2\beta}{\tan\beta} = \frac{\sec^2\beta}{\tan\beta}$$

$$= \frac{2}{2\cos\beta\sin\beta} = 2\,\text{cosec}\,2\beta$$

$$\therefore \quad \sec 2\alpha = \sec\left(\frac{\pi}{2} - 2\beta\right) \Rightarrow 2\alpha = 2n\pi \pm \left(\frac{\pi}{2} - 2\beta\right)$$

Taking +ve sign, we have

$$2(\alpha+\beta) = 2n\pi + \frac{\pi}{2}$$

$$\Rightarrow \quad \alpha + \beta = n\pi + \frac{\pi}{4},\ n \in I$$

For $n = 0, \alpha + \beta = \frac{\pi}{4}$

48. For any two statements p and q, $\sim(p \vee q) \vee (\sim p \wedge q)$ is logically equivalent to

(a) p (b) $\sim p$ (c) q (d) $\sim q$

Sol. *(b)* $\sim(p \vee q) \vee (\sim p \wedge q)$

$\equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$

$\equiv \sim p \wedge (\sim q \vee q) = \sim p$

49. The solution of the differential equation $xdy - ydx - \sqrt{x^2+y^2}\,dx = 0$ is

(a) $y - \sqrt{x^2+y^2} = cx^2$ (b) $y + \sqrt{x^2+y^2} = cx^2$

(c) $y + \sqrt{x^2+y^2} = cy^2$ (d) $x - \sqrt{x^2+y^2} = cy^2$

Sol. *(b)* Given, $\frac{dy}{dx} = \frac{\sqrt{x^2+y^2}+y}{x}$

Put $y = vx$

$$\Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\therefore \quad v + x\frac{dv}{dx} = \frac{\sqrt{x^2+v^2x^2}+vx}{x}$$

$$\Rightarrow \quad \frac{dv}{\sqrt{1+v^2}} = \frac{dx}{x}$$

$$\Rightarrow \quad \log(v + \sqrt{1+v^2}) = \log x + \log c \qquad \text{[integrating]}$$

$$\Rightarrow \quad \log\left(\frac{y}{x} + \sqrt{1+\frac{y^2}{x^2}}\right) = \log cx$$

$$\Rightarrow \quad y + \sqrt{x^2+y^2} = cx^2$$

50. If A and B are independent events of a random experiment such that $P(A \cap B) = \frac{1}{6}$ and $P(\bar{A} \cap \bar{B}) = \frac{1}{3}$, then $P(A)$ is equal to

(a) $\frac{1}{4}$ (b) $\frac{1}{3}$ (c) $\frac{5}{7}$ (d) $\frac{2}{3}$

Sol. *(b)* Since, A and B are independent events.

$$\therefore \quad P(A)P(B) = \frac{1}{6} \text{ and } P(\bar{A})P(\bar{B}) = \frac{1}{3}$$

$$\Rightarrow \quad [1 - P(A)][1 - P(B)] = \frac{1}{3}$$

$$\Rightarrow \quad 1 - [P(A) + P(B)] + P(A)P(B) = \frac{1}{3}$$

$$\Rightarrow \quad 1 + \frac{1}{6} - \frac{1}{3} = P(A) + P(B)$$

$$\Rightarrow \quad P(A) + P(B) = \frac{5}{6}$$

$$\Rightarrow \quad P(A) = \frac{1}{2}, P(B) = \frac{1}{3} \text{ or } P(A) = \frac{1}{3}, P(B) = \frac{1}{2}$$

ENGLISH LANGUAGE AND COMPREHENSION

Directions (Q. Nos. 51-54) *Read the given passage carefully and answer the questions given below.*

It is a commonly held belief that quality and productivity are a function of technology or a set of new equipment. No doubt these are essential, but they alone are not sufficient for bringing about improvements in productivity or quality. It is the men and women behind the machines and the people who manage the technology who are critical in bringing about these improvements. It has been a strange paradox of India's economic development that even though people are our most abundant resource, they have so far either been neglected or treated as liabilities rather than as assets. Part of the reason for this has been outdated labour laws which have been a deterrent for industrialists and employers, leading them to establish capital-intensive rather than labour-intensive operations. The other reason has been a confrontationist attitude, both on the part of labour as well as managements.

A change must come about in both these factors, outside representation and leadership of unions etc need to change. At the same time, the attitude of confrontation must change to one of cooperation and active collaboration.

51. Which of the following arguments has been emphasised in the paragraph?

(a) Only technology or a new set of equipment can improve quality and productivity

(b) Only management behind any type of machines can improve quality and productivity

(c) By managing the new technology, labour can bring about improvements in quality and productivity

(d) Indian labour and management is neither quality nor productivity conscious

Sol. *(c)* If we are able to teach the labour new technology, improvement in quality and productivity can be brought about.

52. India's strange contradiction of development is

(a) people are resourceful but new equipment is not given to them

(b) people are resourceful but they are neglected

(c) labour is not earnest and therefore it is no longer a liability

(d) labour is inefficient but still it is pampered

Sol. *(b)* It is an irony that people of India are very resourceful but they are most often neglected.

53. Capital-intensive operations can lead to

(a) strict labour laws (b) new labour laws

(c) too many labour laws (d) irrelevant labour laws

Sol. *(d)* Irrelevant labour laws. A company's capital expenses are generally judged in relation to its labour expenses, which can lead to irregular labour laws.

54. Which of the following statements on confrontation between labour and management is false?

(a) Too much government interference between labour and management

(b) Conflicting attitude of labour and management

(c) Establishment of capital-intensive industries

(d) Neglect of labour-intensive operations

Sol. *(a)* Too much governmental interference between labour and management.

Directions (Q. Nos. 55-56) *Find out whether there is any grammatical error in the sentences given below. If there is no error, mark the option (d) as your answer.*

55. The flock of lions (a)/roamed about (b)/ fearlessly in the jungle. (c)/No error (d)

Sol. Part (a) has error of use of correct noun, remove flock and write herd.

E.g. For a group of lions we always say **herd** not flock.

56. My brother in laws (a)/who live in Pune (b)/have come to stay with us. (c)/No error (d)

Sol. Part (a) has error of use of plural. We will put plural in 'brothers' not in 'law'.

Directions (Q. Nos. 57-58) *Choose the correct synonym of the given word, out of the four alternatives given.*

57. Pacified

(a) Threatened (b) Pleased

(c) Reprimanded (d) Quietened

Sol. *(d)* is correct synonym as other options do not fit here. Both words mean silenced or satisfied.

E. g. The bottle of milk **pacified** the crying baby.

58. Redundant

(a) Unwilling (b) Surplus

(c) Wrong (d) Mislabel

Sol. *(b)* Surplus is synonym of redundant.

E. g. Eight permanent staff were made **redundant**.

59. Choose the correct meaning of the idiom

To take with a grain of salt

(a) to take with some reservation

(b) to take with total disbelief

(c) to take whole-heartedly

(d) to take seriously

Sol. *(a)* is correct

E. g. I've seen the article which I take **with a grain of salt**.

60. In the following question, a paragraph consists of six sentences, in which first and last sentences S_1 and S_6 are fixed. The middle four sentences are jumbled up labelled P, Q, R, S. Choose the correct sequence of these four sentences.

S_1: The department has initiated steps

P. from the corporate sector

Q. to evolve appropriate schemes

R. and financial institutions for

S. for mobilising investment

S_6 the development of waste lands

(a) PRSQ (b) QPSR

(c) QSPR (d) RPSQ

Sol. *(c)* QSPR

The full sentence will be

The department has initiated steps to evolve appropriate schemes for mobilising investment from the corporate sector and financial institutions for the development of wastelands.

COMPUTER AWARENESS

61. The software that is used for file manipulations, managing directories and subdirectories, programming and accounts setups is known as
(a) Graphic Software (b) Operating System
(c) Application Software (d) Programming Language

Sol. *(c)* Application software is a program or group of programs that is used for file manipulations, managing directories and subdirectories, programming and accounts setups.

62. Which was the first Intel processor introduced?
(a) 3080 (b) 4004
(c) 8080 (d) 8086

Sol. *(b)* The 4004 was the first commercially available computer processor.
The 4004 microprocessor is one of 4 chips constituting MCS-4 chip set which includes the 4001 ROM, 4002 RAM and 4003 shift register.

63. Which of the following is true about presentation layer?
(a) Provide service that directly support the end users of the network.
(b) Maintaining and terminating a dialogue or a session between two end users.
(c) Provide facility to convert message data into a meaningful form.
(d) Responsible for transmitting raw bit streams between two nodes.

Sol. *(c)* The presentation layer provides facilities to convert message data into a form which is meaningful to the communicating application layer entities.

64. Which of the following field of the TCP header tells how many bytes may be sent starting at the byte acknowledged?
(a) TCP header length
(b) Window size
(c) Acknowledgement number
(d) Urgent pointer

Sol. *(b)* The window size field tells how many bytes may be sent starting at the byte acknowledged.

65. The Artificial Intelligence is concerned with designing intelligent computer systems that exhibit intelligent characteristics expressed by
(a) functional behaviour (b) human behaviour
(c) human brain (d) statistical analysis

Sol. *(b)* The Artificial Intelligence is concerned with designing intelligent computer system and this type of computers are more expressed by human behaviour.

66. Which was the world's first mini computer and when was it introduced?
(a) PDP-1, 1958 (b) IBM system/36, 1960
(c) PDP-II, 1961 (d) VAX-11/780, 1962

Sol. *(a)* The PDP-1 was the first computer in digital equipment corporation's PDP series. It was also the original hardware for playing history's first game on a mini computer.

67. Cloud computing is an abstraction based on the notion of pooling physical resources and presenting them as a ..... resource.
(a) real (b) virtual
(c) cloud (d) None of these

Sol. *(b)* Cloud computing is a new model for virtual resources, for staging applications, and for platform independent user access to services.

68. Term that is used for stationary or mobile wireless station and also have optional central base station is called
(a) point-to-point (b) multi point
(c) network point (d) access point

Sol. *(d)* In mobile networking, a Wireless Access Point (WAP) is a networking hardware device that allows a Wi-Fi compliant device to connect to a wired network.

69. Which of the following machine was not invented by Charles Babbage?
(a) Tabulating Machine (b) Analytical Engine
(c) Difference Engine (d) Both (c) and (d)

Sol. *(a)* Charles Babbage, credited deservedly as father of computer, the world renowned inventor of Difference Engine and Analytical Engine.

70. In which topology, if there are n devices in a network, each device has $n-1$ ports for cables?
(a) Mesh (b) Star
(c) Bus (d) Ring

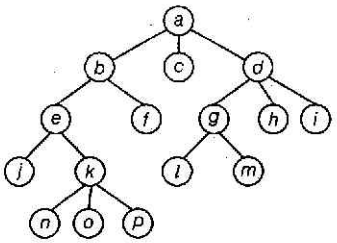
Sol. *(a)* A mesh topology is a network topology in which each node relays data for the network.

71. A program that secretly takes over another Internet attached computer and then uses that computer to launch attacks.
(a) Worm (b) Zombie
(c) Virus (d) Trap doors

Sol. *(b)* A zombie is a computer connected to the Internet that has been comprised by a hacker, computer virus or trojan horse.

72. Consider the In-order and Post-order traversals of a tree as given below:
In-order : j e n k o p b f a c l g m d h i
Post-order : j n o p k e f b c l m g h i d a
The pre-order traversal of the tree shall be
(a) a b f e j k n o p c d g l m h i
(b) a b c d e f j k n o p g l m h i
(c) a b e j k n o p f c d g l m h i
(d) j e n o p k f b c l m g h i d a

Sol. *(c)*



73. Network layer at source is responsible for creating a packet from data that coming from another

(a) data (b) link (c) IP (d) protocol

Sol. *(d)* The network layer is responsible for routing, which is moving packet/creating a packet from data that coming from another protocol.

74. What is the name for information sent from robot sensors to robot controllers?

(a) Temperature (b) Pressure
(c) Feedback (d) Signal

Sol. *(c)* Feedback sensors are used to detect the actuator's output, so that the control system can correct for external factor.

75. A subset of data in a data warehouse in the form of summary data, related to a particular department or business function

(a) meta data (b) archive data
(c) data mart (d) operational data store

Sol. *(c)* A data mart is a subset of the data warehouse that is usually oriented to specific business line or team.

76. What property of the files prevents sharing of files and directories?

(a) Tree structure (b) One level structure
(c) Two level structure (d) Length

Sol. *(a)* Tree structure is one of the property of the file which prevents sharing of files and directories.

77. Devices that provide the connectivity to a WiMAX network are known as

(a) subscriber stations (b) base stations
(c) gateway (d) None of these

Sol. *(a)* WiMAX is a family of wireless communications standard. After network entry is allowed, the subscriber station is allocated an access slot by the base station.

78. Consider the following statements in $a=4$, $b=3$, $c=0$; $c=++a---b+a++---b+b++$;

What will be the value of a, b and c after the statement?

(a) 7, 2, 8 (b) 5, 2, 10
(c) 6, 2, 9 (d) 4, 2, 8

Sol. *(c)* 6, 2, 9

79. Determine the wrong statement about malloc() function in C.

(a) It used to allocate space in memory during the execution of the program.
(b) It does not initialise the memory allocated during execution.
(c) It initialises the allocated memory to zero.
(d) None of the above

Sol. *(c)* Malloc() couldn't able to allocate requested amount of memory.

80. A combinational circuit that converts binary information from n input lines of a maximum of 2^n unique output lines

(a) subtractor (b) decoder
(c) adder (d) multiplexer

Sol. *(b)* A decoder is a circuit that changes a code into a set of signals. It converts binary information from n input lines of a maximum of 2^n unique output lines.

LOGICAL AND ANALYTICAL ABILITY

Directions (Q. Nos. 81-82) *Each question given below consists of a statement, followed by two arguments numbered* I *and* II. *You have to decide which of the arguments is a strong argument and which is a weak argument?*

Give answer
(a) if only argument I is strong
(b) if only argument II is strong
(c) if neither I nor II is strong
(d) if both I and II are strong

81. Statement Should young entrepreneurs be encouraged?

Arguments

I. Yes, they will help in the industrial development of the country.

II. Yes, they will reduce the burden on the employment market.

Sol. *(d)* It is very clear that encouragement to the young entrepreneurs will open up the fields for setting up of new industries. Therefore, it will help in industrial development. Consequently, more job opportunities will be created. Thus, both the arguments are strong.

82. **Statement** Should luxury hotels be banned in India?

Arguments

I. Yes, these are places from where international criminals operate.

II. No, affluent foreign tourists will have no place to stay.

Sol. ***(b)*** The luxury hotels are symbols of country's development and a place for staying the affluent foreign tourists. So, Argument II is a strong one Argument I is a week argument because ban on luxury hotels is not a way to end the international criminals.

83. In a certain code language, 'CURATIVE' is written as 'BSVDDUHS'. How 'STEAMING' is to be written in the same code language?

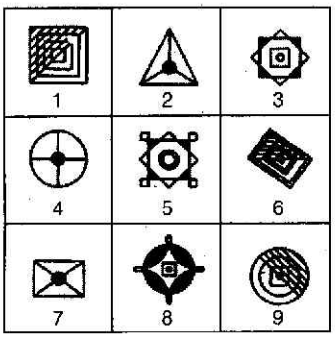
(a) BFUTFMHL (b) TUFBFMHL
(c) BFUTLHMF (d) BFUTHOJN

Sol. ***(a)*** As, C U R A T I V E → +1 → B S V D D U H S

Similarly, S T E A M I N G → +1 → B F U T F M H L

Directions (Q. Nos. 84-85) *In the following questions, group the given figures into three classes using a figure only once.*

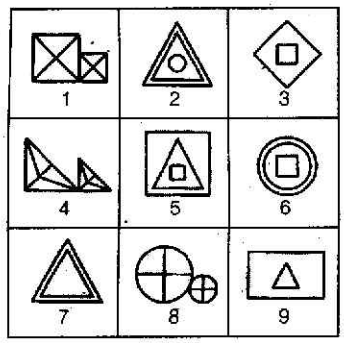
84.



(a) (1, 6, 9), (2, 5, 7), (4, 8, 3) (b) (1, 6, 9), (2, 4, 8), (3, 5, 7)
(c) (1, 3, 5), (2, 6, 7), (4, 8, 9) (d) (1, 6, 9), (2, 4, 7), (3, 5, 8)

Sol. ***(d)*** 1, 6 and 9 form a group of figures in which hall of the figure contains parallel straight lines. 2, 4 and 7 form a group of figures which are divided into equal parts and a dark circle is present at their centre. 3, 5 and 8 form a group of figures in which similar design pattern is followed.

85.



(a) (1, 3, 7), (2, 4, 6), (5, 8, 9)
(b) (1, 4, 6), (2, 5, 7), (3, 8, 9)
(c) (1, 4, 8), (2, 5, 6), (3, 7, 9)
(d) (1, 4, 8), (2, 7, 9), (3, 5, 6)

Sol. ***(c)*** 1, 4 and 8 contain similar figures both divided into four parts and attached to each other.

2, 5 and 6 contain three figures (two of which are similar) placed one inside the other.

3, 7 and 9 contain one figure inside the other which may or may not be similar.

Directions (Q. Nos. 86-87) *In each of the following questions, arrange the words in a meaningful, logical order then select the appropriate sequence from the alternatives given below each of the groups of words.*

86. 1. Trillion 2. Thousand
3. Billion 4. Hundred
5. Million

(a) 1, 2, 4, 3, 5 (b) 1, 5, 3, 2, 4
(c) 4, 2, 3, 5, 1 (d) 4, 2, 5, 3, 1

Sol. ***(d)*** All the words represent the counting numbers and their increasing order is given as below

Hundred → Thousand → Million → Billion → Trillion

This order is given in option (d), i.e. 4, 2, 5, 3, 1.

87. 1. Country 2. Furniture
3. Forest 4. Wood
5. Trees

(a) 1, 3, 5, 4, 2 (b) 1, 4, 3, 2, 5
(c) 2, 4, 3, 1, 5 (d) 5, 2, 3, 1, 4

Sol. ***(a)*** From the above words, it is deduced that a country contains forests, a forest has trees, trees give wood that is used to make furniture. Hence, the correct sequence is (a).

Directions (Q. Nos. 88-90) *Complete the series by choosing the correct option.*

88. 101, 100, ? 87, 71, 46

(a) 92 (b) 88
(c) 89 (d) 96

Sol. ***(d)*** The pattern is as follows

101 100 (96) 87 71 46
$-(1)^2$ $-(2)^2$ $-(3)^2$ $-(4)^2$ $-(5)^2$

Missing number $= 100 - (2)^2 = 100 - 4 = 96$

89. 4, ?, 144, 400, 900, 1764

(a) 25 (b) 36
(c) 49 (d) 100

Sol. ***(b)*** The pattern is as follows

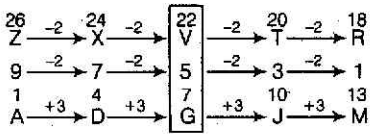
4 (36) 144 400 900 1764
$(2)^2$ $(6)^2$ $(12)^2$ $(20)^2$ $(30)^2$ $(42)^2$
+4 +6 +8 +10 +12

$\therefore ? = 36$

90. Z9A, X7D, ?, T3J, R1M

(a) W6F (b) S3H
(c) G9V (d) V5G

Sol. *(d)* Pattern of given alpha-numeric series is as follows



$\therefore$? = V5G

Directions (Q. Nos. 91-92) *Read the following information carefully and answer the questions that follow*

I. P + Q means P is the father of Q.

II. P – Q means P is the wife of Q.

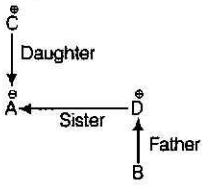
III. P × Q means P is the brother of Q.

IV. P + Q means P is the daughter of Q.

91. If A + C + D + B, then which of the following statement is true?

(a) A is the daughter of B (b) B is the aunt of A
(c) A is the aunt of B (d) A is the mother of B

Sol. *(c)* From the given information following family diagram can be drawn

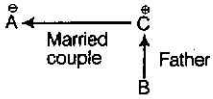


So, it is clear that A is the aunt of B.

92. If A – C + B, then which of the following statements is true?

(a) A is the daughter of B (b) B is the aunt of A
(c) A is the mother of B (d) A is the aunt of B

Sol. *(c)* From the given information, following family diagram can be drawn



Now, it is clear that A is the mother of B.

Directions (Q. Nos. 93-94) *In each of the questions below are given three statements followed by four conclusions numbered* I, II, III *and* IV. *You have to take the given statements to be true even, if they seem to be at variance from commonly known facts. Read all the conclusions and then decide which of the given conclusion(s) logically follow(s) from the given statements disregarding commonly known facts.*

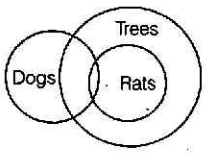
93. Statements Some dogs are rats.
All rats are trees.
Some trees are not dogs.

Conclusions

I. Some trees are dogs.
II. All dogs are trees.
III. All rats are dogs.
IV. No tree is dog.

(a) None follows (b) Only I follows
(c) I and II follow (d) II and III follow

Sol. *(b)*



Conclusions

I. Some trees are dogs. (✔)
II. All dogs are trees. (✘)
III. All rats are dogs. (✘)
IV. No tree is dog. (✘)

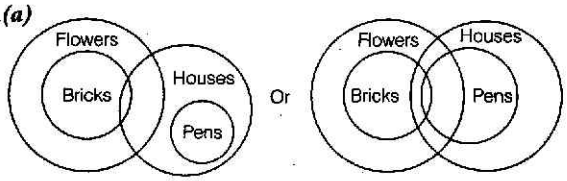
94. Statements All bricks are flowers.
Some houses are flowers.
All pens are houses.

Conclusions

I. Some houses are bricks.
II. Some pens are flowers.
III. Some flowers are bricks.
IV. No pen is flower.

(a) Either I or II and III follow
(b) Either II or IV and I follow
(c) Either I or III and IV follow
(d) None follows

Sol. *(a)*



Conclusions

I. Some houses are bricks. (✔)
II. Some pens are flowers. (✔)
III. Some flowers are bricks. (✔)
IV. No pen is flower. (✘)

Directions (Q. Nos. 95-98) *Examine the information given in the following paragraph and answer the items that follow.*

Guest lectures on five subjects *viz.* Economics, History, Statistics, English and Mathematics have to be arranged in a week from Monday to Friday. Only one lecture can be arranged on each day. Economics cannot be scheduled on Tuesday. Guest faculty for History is available only on Tuesday. Mathematics lecture has to be scheduled immediately after the day of Economics lecture. English lecture has to be scheduled immediately before the day of Economics lecture.

95. Which lecture is scheduled on Monday?
(a) History (b) Economics
(c) Mathematics (d) Statistics

96. Which lecture is scheduled between Statistics and English?
(a) Economics (b) History
(c) Mathematics (d) No lecture

97. Which lecture is the last one in the week?
(a) History (b) English
(c) Mathematics (d) Economics

98. Which lecture is scheduled on Wednesday?
(a) Statistics
(b) Economics
(c) English
(d) History

Sol. (Q. Nos. 95-98) *Information given in the passage can be arranged in the tabular form like this*

Days	Monday	Tuesday	Wednesday	Thursday	Friday
Subject	Statistics	History	English	Economics	Maths

95. *(d)* Statistics lecture is scheduled on Monday.

96. *(b)* From the above table History is scheduled between Statistics and English.

97. *(c)* Mathematics lecture is scheduled on the last day of the week.

98. *(c)* English lecture is scheduled on Wednesday.

Directions (Q. Nos. 99-100) *In each of the following questions, a related pair of words is followed by four pairs of words or phrases. Select the pair that best expresses a relationship similar to the one expressed in the question pair.*

99. Dubious: Certain
(a) Hot : Angry (b) Cold : Warm
(c) Long : Elongated (d) Short : Dawarfish

Sol. *(b)* Dubious is the antonym of Certain, similarly Cold is the antonym of Warm.

100. Indolence : Beaver
(a) Elegence : Peacock (b) Ferocity : Lomb
(c) Passivity : Cow (d) Joviality : Hyena

Sol. *(a)* Beaver is known for its Indolence, similarly Peacock is known for its beauty or 'Elegence'.