# Jamia Millia Islamia

## Master of Computer Application (MCA)

## Entrance Examination

## Solved Paper 2017

**Time Allowed : 2.5 hrs** **Max. Marks : 170**

**Instructions for Candidates**

- *(i)* All questions in this paper are of objective type.
- *(ii)* Their is a total of 100 questions.
- *(iii)* (Q. Nos. 1-50) consist of Mathematics.
- *(iv)* (Q. Nos 51-60) consist of English Language and Comprehension.
- *(v)* (Q. Nos. 61-80) consist of Computer Awareness.
- *(vi)* (Q. Nos. 81-100) consist of Logical and Analytical Ability.

## MATHEMATICS

**1.** If $\theta = \tan^{-1}\frac{1}{1+2} + \tan^{-1}\frac{1}{1+(2)(3)} + \tan^{-1}\frac{1}{1+(3)(4)} + \ldots + \tan^{-1}\frac{1}{1+n(n+1)}$, then $\tan\theta$ is equal to

(a) $\frac{n}{n+1}$ (b) $\frac{n+1}{n+2}$ (c) $\frac{n}{n+2}$ (d) $\frac{n-1}{n+2}$

**Ans.** (c) $\theta = \tan^{-1}\frac{1}{1+2} + \tan^{-1}\frac{1}{1+(2)(3)} + \tan^{-1}\frac{1}{1+(3)(4)} + \ldots + \tan^{-1}\frac{1}{1+n(n+1)}$

$= \tan^{-1}\frac{2-1}{1+1.2} + \tan^{-1}\frac{3-2}{1+(2)(3)} + \tan^{-1}\frac{4-3}{1+(3)(4)} + \ldots + \tan^{-1}\frac{(n+1)-n}{1+n(n+1)}$

$= (\tan^{-1}2 - \tan^{-1}1) + (\tan^{-1}3 - \tan^{-1}2) + (\tan^{-1}4 - \tan^{-1}3) + \ldots + (\tan^{-1}(n+1) - \tan^{-1}n)$

$= \tan^{-1}(n+1) - \tan^{-1}1$

$= \tan^{-1}\frac{(n+1)-1}{1+1(n+1)} = \tan^{-1}\frac{n}{n+2} \Rightarrow \tan\theta = \frac{n}{n+2}$

**2.** The number of solutions for

$$\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2} \text{ is}$$

(a) zero (b) one
(c) two (d) infinite

**Ans.** (c) $\because \tan^{-1}\sqrt{x(x+1)} = \cos^{-1}\frac{1}{\sqrt{x^2+x+1}}$

$\therefore \tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$

Given, $\cos^{-1}\frac{1}{\sqrt{x^2+x+1}} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$

which holds if $\frac{1}{\sqrt{x^2+x+1}} = \sqrt{x^2+x+1}$

$\left(\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right)$

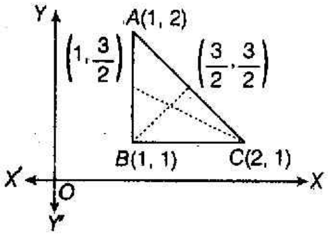
$\Rightarrow x^2+x+1 = 1 \Rightarrow x(x+1) = 0 \Rightarrow x = 0 \text{ or } -1$

**3.** Let $ABC$ be an isosceles triangle with $AB = BC$. If base $BC$ is parallel to $X$-axis and $m_1, m_2$ are slopes of medians drawn through the angular points $B$ and $C$. then

(a) $m_1 \cdot m_2 = \frac{1}{2}$ (b) $m_1 + m_2 = 0$

(c) $m_1 - m_2 = \frac{3}{2}$ (d) $(m_1 - m_2)^2 + 2m_1m_2 = 0$

**Ans.** (c)



From above figure $m_1 = 1; m_2 = -\frac{1}{2}$

But $m_1 m_2 = -\frac{1}{2}$

$$m_1 + m_2 = \frac{1}{2}$$

$$m_1^2 + m_2^2 = (m_1 + m_2)^2 - 2m_1m_2 = \frac{1}{4} + 1$$

$$m_1^2 + m_2^2 = \frac{5}{4}$$

$$(m_1 - m_2)^2 = (m_1 + m_2)^2 - 4m_1m_2$$

$$= \frac{1}{4} - 4(-1/2) = \frac{1}{4} + 2 = \frac{9}{4}$$

$$m_1 - m_2 = \frac{3}{2}$$

So, option (c) is correct.

**4.** The value of $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx$ is

(a) $\frac{\pi^2}{3}$ (b) $\frac{\pi^2}{4}$

(c) $\frac{\pi^2}{6}$ (d) $\frac{\pi^2}{2}$

**Ans.** (b) $I = \int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx = \int_0^\pi \frac{(\pi - x) \sin x}{1 + \cos^2 x} dx$

$$\left[\because \int_0^a f(x)\, dx = \int_0^a (a - x)\, dx\right]$$

$$2I = \pi \int_0^\pi \frac{\sin x}{1 + \cos^2 x} dx$$

$\int_0^{2a} f(x)\, dx = 2\int_0^a f(x)\, dx$, if $f(2a - x) = f(x)$

$$= 2\pi \int_0^{\pi/2} \frac{\sin x}{1 + \cos^2 x} dx$$

$$\Rightarrow \quad I = \pi \int_0^{\pi/2} \frac{\sin x}{1 + \cos^2 x} dx$$

Let $\cos x = z \Rightarrow -\sin x\, dx = dz$

$$\therefore \quad I = \pi \int_1^0 -\frac{dz}{1 + z^2} = \pi \int_0^1 \frac{dz}{1 + z^2}$$

$$= \pi\, [\tan^{-1} z]_0^1 = \pi \left(\frac{\pi}{4}\right) = \frac{\pi^2}{4}$$

**5.** The vector $\mathbf{B} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{k}}$ is to be written as the sum of a vector $\mathbf{B}_1$ parallel to $\mathbf{A} = \hat{\mathbf{i}} + \hat{\mathbf{j}}$ and a vector $\mathbf{B}_2$ perpendicular to $\mathbf{A}$, then $\mathbf{B}_1$ is

(a) $\frac{3}{2}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ (b) $\frac{2}{3}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

(c) $\frac{1}{2}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ (d) None of these

**Ans.** (a) Given, $\mathbf{B}_1 = \lambda \mathbf{A} = \lambda\, (\hat{\mathbf{i}} + \hat{\mathbf{j}})$

$$\mathbf{B}_2 . \mathbf{A} = 0$$

$\Rightarrow (\mathbf{B} - \mathbf{B}_1) . \mathbf{A} = 0$ $[\because \mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2$ (given)$]$

$$\Rightarrow \quad \mathbf{B} . \mathbf{A} = \mathbf{B}_1 . \mathbf{A}$$

$$(3\hat{\mathbf{i}} + 4\hat{\mathbf{k}}) . (\hat{\mathbf{i}} + \hat{\mathbf{j}}) = \lambda(\mathbf{A} . \mathbf{A}) = \lambda(1 + 1)$$

$$\Rightarrow \quad 3 = 2\lambda \quad \Rightarrow \lambda = \frac{3}{2}$$

$$\therefore \quad \mathbf{B}_1 = \frac{3}{2}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$$

**6.** *A* and *B* throw a die in succession to win a bet with *A* starting first. Whoever throws '1' first wins an amount of ₹110. What are the respective expectations of *A* and *B*?

(a) ₹ 70 and ₹ 40 (b) ₹ 60 and ₹ 50

(c) ₹ 75 and ₹ 35 (d) None of these

**Ans.** (b) $P(1) = \frac{1}{6}; P(\bar{1}) = \frac{5}{6}$

$$P(A \text{ wins}) = \frac{1}{6} + \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} + \ldots$$

$$= \frac{1}{6}\left[1 + \frac{25}{36} + \left(\frac{25}{36}\right)^2 + \ldots\right] = \frac{\frac{1}{6}}{1 - \frac{25}{36}} = \frac{1/6}{11/36} = \frac{6}{11}$$

$$P(B \text{ wins}) = 1 - P(A \text{ wins})' = 1 - \frac{6}{11} = \frac{5}{11}$$

$$E(\text{amount of } A) = 110 \times \frac{6}{11} = ₹\, 60$$

$$E(\text{amount of } B) = 110 \times \frac{5}{11} = ₹\, 50$$

**7.** The probability that a man who is 85 yr old will die before attaining the age of 90 yr is 1/3. $A_1, A_2, A_3$ and $A_4$ are four persons who are 85 yr old. The probability that $A_1$ will die before attaining the age of 90 yr and will be the first to die is

(a) $\frac{65}{81}$ (b) $\frac{13}{81}$

(c) $\frac{65}{324}$ (d) $\frac{13}{108}$

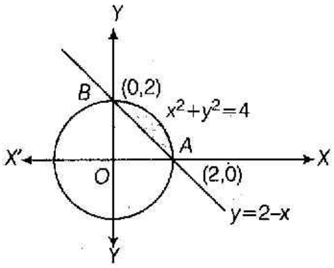
**Ans.** (c) Required probability = *P* (atleast one person dies before 90 yr) × *P* (first person to die is $A_1$)

$$= \left[1 - \left(\frac{2}{3}\right)^4\right] \times \left[\frac{3!}{4!}\right] = \left(1 - \frac{16}{81}\right) \times \frac{1}{4} = \frac{65}{324}$$

**8.** The smaller of the areas bound by $y = 2 - x$ and $x^2 + y^2 = 4$, is

(a) $(\pi - 1)$ sq units
(b) $(\pi - 2)$ sq units
(c) $(2\pi - 1)$ sq units
(d) $(2\pi - 2)$ sq units

**Ans.** (b)



From the above figure, shaded area is the smaller area bounded by the given line and circle.

$$= \frac{1}{4}(\text{Area of circle}) - \text{Area of } \Delta AOB$$

$$= \frac{1}{4}\pi(2)^2 - \frac{1}{2} \times 2 \times 2 = (\pi - 2) \text{ sq units}$$

**9.** The number of distinct integral values of '$a$' satisfying the equation $2^{2a} - 3(2^{a+2}) + 2^5 = 0$ is

(a) 0 (b) 1 (c) 2 (d) 3

**Ans.** (c) $2^{2a} - 3(2^{a+2}) + 2^5 = 0$

Let $y = 2^a$

$\Rightarrow$ $y^2 - 12y + 32 = 0$

$\Rightarrow$ $(y-4)(y-8) = 0$

$\Rightarrow$ $y = 4, 8$

$2^a = 4 \Rightarrow a = 2$

$2^a = 8 \Rightarrow a = 3$

Thus, the number of integrals satisfying $a$ is 2 (two).

**10.** If $y = f(x)$ is an odd and differentiable function defined on $(-\infty, \infty)$ such that $f'(3) = -2$, then $f'(-3)$ equals to

(a) 4 (b) 2 (c) – 2 (d) 0

**Ans.** (c) As $f(x)$ is an odd function, so we have

$f(x) = -f(-x)$

$\Rightarrow$ $f'(x) = f'(-x)$

$\Rightarrow$ $f'(3) = f'(-3) = -2$

**11.** The straight lines $\frac{x}{a} - \frac{y}{b} = k$ and $\frac{x}{a} + \frac{y}{b} = \frac{1}{k}, k \neq 0$ meet on

(a) a parabola
(b) an ellipse
(c) a hyperbola
(d) a circle

**Ans.** (c) $\frac{x}{a} - \frac{y}{b} = k$ and $\frac{x}{a} + \frac{y}{b} = \frac{1}{k}$ is jointly satisfied by

$$\left(\frac{x}{a} - \frac{y}{b}\right)\left(\frac{x}{a} + \frac{y}{b}\right) = (k)\left(\frac{1}{k}\right)$$

$\Rightarrow$ $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, which is a hyperbola.

**12.** Let $A$ and $B$ be two events such that

$$P(\overline{A \cup B}) = \frac{1}{6}, P(A \cap B) = \frac{1}{4} \text{ and } P(\overline{A}) = \frac{1}{4}.$$

Then, events $A$ and $B$ are

(a) independent but not equally likely.
(b) mutually exclusive and independent.
(c) equally likely and mutually exclusive.
(d) equally likely but not independent.

**Ans.** (a) $P(\overline{A \cup B}) = \frac{1}{6} \Rightarrow P(A \cup B) = 1 - \frac{1}{6} = \frac{5}{6}$

$P(\overline{A}) = \frac{1}{4}$

$\Rightarrow$ $P(A) = 1 - \frac{1}{4} = \frac{3}{4}$

$P(B) = P(A \cup B) + P(A \cap B) - P(A) = \frac{5}{6} + \frac{1}{4} - \frac{3}{4} = \frac{1}{3}$

$\Rightarrow$ $P(A) = \frac{3}{4}; P(B) = \frac{1}{3}$

$\because$ $P(A \cap B) = P(A) \cdot P(B)$

$\frac{1}{4} = \frac{3}{4} \cdot \frac{1}{3} = \frac{1}{4}$

$\therefore A$ and $B$ are independent but not equally likely.

**13.** If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then $I + A + A^2 + \ldots \infty$ equals to

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
(b) $\begin{bmatrix} -1 & -2 \\ -3 & -4 \end{bmatrix}$
(c) $\begin{bmatrix} \frac{1}{2} & -\frac{1}{3} \\ -\frac{1}{2} & 0 \end{bmatrix}$
(d) $\begin{bmatrix} -\frac{1}{4} & \frac{1}{3} \\ \frac{1}{2} & 0 \end{bmatrix}$

**Ans.** (c) Let $\lambda$ be an eigen value of given matrix $A$.

Now, $1 + \lambda + \lambda^2 + \ldots \infty = \frac{1}{1 - \lambda}$

$\Rightarrow$ $(1 - \lambda)(1 + \lambda + \lambda^2 + \ldots \infty) = 1$

By Cayley-Hamilton's theorem, "Every square matrix satisfy its characteristic equation".

$(I - A)(I + A + A^2 + \ldots \infty) = I$

$\Rightarrow$ $I + A + A^2 + \ldots + \infty = (I - A)^{-1}$

$$= \begin{bmatrix} 0 & -2 \\ -3 & -3 \end{bmatrix}^{-1} = -\frac{1}{6}\begin{bmatrix} -3 & 2 \\ 3 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{3} \\ -\frac{1}{2} & 0 \end{bmatrix}$$

**14.** $a, b, c$ are non-coplanar unit vectors such that $a \times (b \times c) = \frac{b + c}{\sqrt{2}}$, then the angle between $a$ and $b$ is

(a) $\frac{\pi}{4}$
(b) $\frac{3\pi}{4}$
(c) $\frac{\pi}{2}$
(d) $\pi$

**Ans.** (b) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\,\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$ ...(i)

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \frac{\mathbf{b} + \mathbf{c}}{\sqrt{2}} \qquad ...(ii)$$

From Eqs. (i) and (ii)

$$\mathbf{a} \cdot \mathbf{c} = \frac{1}{\sqrt{2}} \text{ and } \mathbf{a} \cdot \mathbf{b} = -\frac{1}{\sqrt{2}}$$

Let $\theta$ be the angle between **a** and **b**.

$$\Rightarrow \quad |\mathbf{a}| \cdot |\mathbf{b}| \cos\theta = -\frac{1}{\sqrt{2}} \Rightarrow \cos\theta = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \theta = \frac{3\pi}{4}$$

**15.** If $f(x) + f(1-x) = 2$, then the value of

$$f\left(\frac{1}{2001}\right) + f\left(\frac{2}{2001}\right) + \dots\dots + f\left(\frac{2000}{2001}\right) \text{ is}$$

(a) 2000 (b) 2001 (c) 1999 (d) 1998

**Ans.** (a) $f(x) + f(1-x) = 2$

$$\Rightarrow \quad f\left(\frac{1}{2001}\right) + f\left(\frac{2000}{2001}\right) = 2$$

$$\Rightarrow \quad f\left(\frac{2}{2001}\right) + f\left(\frac{1999}{2001}\right) = 2$$

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$$\Rightarrow \quad f\left(\frac{1000}{2001}\right) + f\left(\frac{1001}{2001}\right) = 2$$

Which are 1000 pairs in all.

So, $f\left(\frac{1}{2001}\right) + f\left(\frac{2}{2001}\right) + \dots + f\left(\frac{2000}{2001}\right) = 2000$

**16.** If $y = mx$ bisects the angle between the lines $x^2(\tan^2\theta + \cos^2\theta) + 2xy\tan\theta - y^2\sin^2\theta = 0$ when $\theta = \frac{\pi}{3}$, then the value of $\sqrt{3}\,m^2 + 4m$ is

(a) 1 (b) $\frac{1}{\sqrt{3}}$

(c) $\sqrt{3}$ (d) $7\sqrt{3}$

**Ans.** (c) Equation of angle bisectors of $ax^2 + 2hxy + by^2 = 0$ is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

$\Rightarrow$ Angle bisectors of

$$x^2(\tan^2\theta + \cos^2\theta) + 2xy\tan\theta - y^2\sin^2\theta = 0 \text{ is}$$

$$\frac{x^2 - y^2}{\tan^2\theta + \cos^2\theta + \sin^2\theta} = \frac{xy}{\tan\theta}$$

$$\Rightarrow \quad \frac{x^2 - y^2}{\sec^2\theta} = \frac{xy}{\tan\theta} \qquad [\because \theta = \pi/3]$$

$$\Rightarrow \quad \frac{x^2 - y^2}{4} = \frac{xy}{\sqrt{3}} \qquad ...(i)$$

As $y = mx$ satisfy Eq. (i), so

$$\frac{x^2 - m^2x^2}{4} = \frac{mx^2}{\sqrt{3}} \Rightarrow \frac{1 - m^2}{4} = \frac{m}{\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3} - \sqrt{3}\,m^2 = 4m$$

$$\Rightarrow \quad \sqrt{3}\,m^2 + 4m = \sqrt{3}$$

**17.** A line $L$ has intercepts '$a$' and '$b$' on the coordinate axes. When the axes are rotated through a given angle, keeping the origin fixed, the same line has intercepts '$p$' and '$q$' which of the following statements is true?

(a) $a^2 + b^2 = p^2 + q^2$ (b) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$

(c) $a^2 + p^2 = b^2 + q^2$ (d) $\frac{1}{a^2} + \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{q^2}$

**Ans.** (b) The line $L$ will be $\frac{x}{a} + \frac{y}{b} = 1$ in $xy$-coordinate system.

When the axes are rotated by an angle '$\theta$' in anti-clockwise direction.

$$x' = x\cos\theta + y\sin\theta$$
$$y' = -x\sin\theta + y\cos\theta \qquad ...(i)$$
$$\Rightarrow \quad x = x'\cos\theta - y'\sin\theta$$
$$y = x'\sin\theta + y'\cos\theta$$

$$\Rightarrow \quad \text{Line is } \frac{x'\cos\theta - y'\sin\theta}{a} + \frac{x'\sin\theta + y'\cos\theta}{b} = 1$$

$$\Rightarrow \quad x'\left[\frac{\cos\theta}{a} + \frac{\sin\theta}{b}\right] + y'\left[\frac{\cos\theta}{b} - \frac{\sin\theta}{a}\right] = 1$$

$\Rightarrow$ Intercept $p$ and $q$ are

$$p = \frac{ab}{b\cos\theta + a\sin\theta};\ q = \frac{ab}{a\cos\theta - b\sin\theta} \text{ [given]}$$

$$\Rightarrow \quad \frac{1}{p^2} + \frac{1}{q^2} = \frac{\begin{bmatrix} a^2\sin^2\theta + b^2\cos^2\theta \\ + a^2\cos^2\theta + b^2\sin^2\theta \end{bmatrix}}{a^2b^2} = \frac{a^2 + b^2}{a^2b^2}$$

$$= \frac{1}{a^2} + \frac{1}{b^2}$$

**18.** If $a, b, c$ are the roots of the equation $x^3 - 3px^2 + 3qx - 1 = 0$, then the centroid of the triangle with vertices $\left(a, \frac{1}{a}\right), \left(b, \frac{1}{b}\right)$ and $\left(c, \frac{1}{c}\right)$ is at the point

(a) $(p, q)$ (b) $\left(\frac{p}{3}, \frac{q}{3}\right)$

(c) $(p + q, p - q)$ (d) $(3p, 3q)$

**Ans.** (a) $a$, $b$ and $c$ are roots of $x^3 - 3px^2 + 3qx - 1 = 0$

$$\Rightarrow \quad a + b + c = 3p;\ ab + bc + ca = 3q$$

and $abc = 1$

$$\Rightarrow \quad \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{3q}{abc}$$

$$\Rightarrow \quad \frac{1}{a}+\frac{1}{b}+\frac{1}{c}=3q$$

Now, centroid of triangle with vertices $\left(a, \frac{1}{a}\right), \left(b, \frac{1}{b}\right), \left(c, \frac{1}{c}\right)$ is

$$\left(\frac{a+b+c}{3}, \frac{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}{3}\right)=(p, q)$$

**19.** A letter is known to have come from either TATANAGAR or CALCUTTA. On the envelope, just two consecutive letters, TA, are visible. The probability that the letter has come from CALCUTTA is

(a) 4/11 (b) 1/3

(c) 5/12 (d) None of these

**Ans.** (a) TATANAGAR has 9 letters, so number of ways in which two consecutive letters can be printed will be 8, out of which there are 2 ways in which TA can be printed. Similarly, for CALCUTTA, there are 7 ways of printing two consecutive letters, from which there is only one way to print 'TA'.

Here, required probability $= \dfrac{\frac{1}{7}}{\frac{1}{7}+\frac{2}{8}} = \dfrac{\frac{1}{7}}{\frac{1}{7}+\frac{1}{4}} = \dfrac{4}{11}$

**20.** If $\cos\alpha+\cos\beta=a$, and $\sin\alpha+\sin\beta=b$ and $\theta$ is the Arithmetic Mean between $\alpha$ and $\beta$, then $\sin 2\theta+\cos 2\theta$ is equal to

(a) $\dfrac{(a+b)^2}{(a^2+b^2)}$ (b) $\dfrac{(a-b)^2}{(a^2+b^2)}$

(c) $\dfrac{a^2-b^2}{a^2+b^2}$ (d) None of these

**Ans.** (d) Given, $\cos\alpha+\cos\beta=a$

$$\Rightarrow \quad 2\cos\left(\frac{\alpha+\beta}{2}\right), \cos\left(\frac{\alpha-\beta}{2}\right)=a$$

$$\Rightarrow \quad 2\cos\theta\cos\left(\frac{\alpha-\beta}{2}\right)=a \quad \text{...(i)}$$

$$\left[\because \theta=\frac{\alpha+\beta}{2} \text{(AM of } \alpha, \beta)\right]$$

Also, $\sin\alpha+\sin\beta=b$

$$\Rightarrow \quad 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)=b$$

$$\Rightarrow \quad 2\sin\theta\cos\left(\frac{\alpha-\beta}{2}\right)=b \quad \text{...(ii)}$$

$$\Rightarrow \quad \frac{a}{\cos\theta}=\frac{b}{\sin\theta}=\frac{\sqrt{a^2+b^2}}{\sqrt{\sin^2\theta+\cos^2\theta}}=\sqrt{a^2+b^2}$$

[from Eqs. (i) and (ii)]

$$\Rightarrow \quad \cos\theta=\frac{a}{\sqrt{a^2+b^2}} \text{ and } \sin\theta=\frac{b}{\sqrt{a^2+b^2}}$$

$$\Rightarrow \quad \sin 2\theta=\frac{2ab}{a^2+b^2}; \ \cos 2\theta=\frac{a^2-b^2}{a^2+b^2}$$

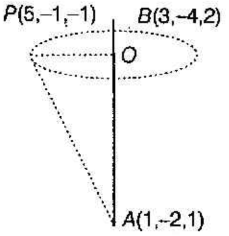
$$\Rightarrow \quad \sin 2\theta+\cos 2\theta=\frac{a^2-b^2+2ab}{a^2+b^2}$$

**21.** A rigid body is rotating at the rate of 3 radians about an axis $AB$, where $A$ and $B$ are the points $(1,-2,1)$ and $(3,-4,2)$. The velocity of the point $P$ at $(5,-1,-1)$ of the body is

(a) $3\hat{\mathbf{i}}+8\hat{\mathbf{j}}+10\hat{\mathbf{k}}$ (b) $\dfrac{3\hat{\mathbf{i}}+8\hat{\mathbf{j}}+10\hat{\mathbf{k}}}{3}$

(c) $\dfrac{2\hat{\mathbf{i}}+2\hat{\mathbf{j}}+\hat{\mathbf{k}}}{3}$ (d) $4\hat{\mathbf{i}}+\hat{\mathbf{j}}+2\hat{\mathbf{k}}$

**Ans.** (b)



$$\mathbf{AB}=2\hat{\mathbf{i}}-2\hat{\mathbf{j}}+\hat{\mathbf{k}}$$

$$\mathbf{AP}=4\hat{\mathbf{i}}+\hat{\mathbf{j}}-2\hat{\mathbf{k}}$$

$$\Rightarrow AB\times AP=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 2 & -2 & 1\\ 4 & 1 & -2\end{vmatrix}=3\hat{\mathbf{i}}+8\hat{\mathbf{j}}+10\hat{\mathbf{k}}$$

$$\Rightarrow \quad V=\frac{AB\times AP}{w}=\frac{3\hat{\mathbf{i}}+8\hat{\mathbf{j}}+10\hat{\mathbf{k}}}{3}$$

**22.** The value of $\int_0^{\pi/2}\dfrac{dx}{1+\tan^3 x}$ is

(a) 0 (b) 1

(c) $\dfrac{\pi}{4}$ (d) $\dfrac{\pi}{2}$

**Ans.** (c) $I=\displaystyle\int_0^{\pi/2}\frac{dx}{1+\tan^3 x}=\int_0^{\pi/2}\frac{dx}{1+\tan^3\left(\frac{\pi}{2}-x\right)}$

$$=\int_0^{\pi/2}\frac{dx}{1+\cot^3 x}$$

$$\Rightarrow \quad 2I=\int_0^{\pi/2}\left(\frac{1}{1+\tan^3 x}+\frac{1}{1+\cot^3 x}\right)dx$$

$$\Rightarrow \quad 2I=\int_0^{\pi/2}dx=\frac{\pi}{2} \quad \Rightarrow I=\frac{\pi}{4}$$

**23.** The integer $n$ for which $\displaystyle\lim_{x\to 0}\frac{(\cos x-1)(\cos x-e^x)}{x^n}$ is a finite non-zero number is

(a) 1 (b) 2

(c) 3 (d) 4

**Ans.** (c) $\displaystyle\lim_{x\to 0}=\frac{(\cos x-1)(\cos x-e^x)}{x^n}$

$$= \lim_{x \to 0} \frac{\left(1 - \frac{x^2}{2!} + \ldots - 1\right)\left(1 - \frac{x^2}{2!} + \ldots - \left(1 + \frac{x}{1!} + \ldots\right)\right)}{x^n}$$

$$= \lim_{x \to 0} \frac{\left(-\frac{x^2}{2!} + \frac{x^4}{4!} + \ldots\right)\left(-\frac{x}{1!} - \frac{2x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} \ldots\right)}{x^n}$$

$$= \lim_{x \to 0} \frac{x^3\left(-\frac{1}{2!} + \frac{x^2}{4!} + \ldots\right)\left(-1 - \frac{2x}{2!} - \frac{x^2}{3!} \ldots\right)}{x^n}$$

Which will be finite non-zero value, if $n = 3$ and the value is $\frac{1}{2}$.

**24.** If $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right) x \in [0, \infty)$ and $x \neq 1$, then $\frac{dy}{dx}$ is equal to

(a) 1 (b) $\frac{x-1}{x+1}$

(c) 0 (d) $\frac{x+1}{x-1}$

**Ans.** (c) $y = \sec^{-1}\left(\frac{x+1}{x-1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$ $\left[\because \sec^{-1} x = \cos^{-1} \frac{1}{x}\right]$

$\Rightarrow y = \cos^{-1}\left(\frac{x-1}{x+1}\right) + \sin^{-1}\left(\frac{x-1}{x+1}\right)$

$\left[\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}\right]$

$\Rightarrow y = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$

**25.** The inverse of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 8 & 2 \end{bmatrix}$ is

(a) $\begin{bmatrix} 10 & -6 & 1 \\ -2 & -1 & 0 \\ -7 & -5 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 10 & -6 & 1 \\ -2 & -1 & 0 \\ -7 & -5 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 10 & -6 & -1 \\ -2 & 1 & 0 \\ -7 & 5 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 10 & -6 & 1 \\ -2 & 1 & 0 \\ -7 & 5 & -1 \end{bmatrix}$

**Ans.** (d) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 8 & 2 \end{bmatrix}$

$|A| = 1(6 - 16) - 1(4 - 6) + 1(16 - 9)$

$= -10 + 2 + 7 = -1$

$$\text{adj}(A) = \begin{bmatrix} -10 & 2 & 7 \\ 6 & -1 & -5 \\ -1 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} -10 & 6 & -1 \\ 2 & -1 & 0 \\ 7 & -5 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|} = \begin{bmatrix} 10 & -6 & 1 \\ -2 & 1 & 0 \\ -7 & 5 & -1 \end{bmatrix}$$

**26.** The differential equation

$$\frac{dy}{dx} + (\tan x)\, y = \cos x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

has the solution

(a) $y = (x + C) \cos x$ (b) $y = (x + C) \sec x$

(c) $y = (x + C) \sin x$ (d) $y = (x + C) \operatorname{cosec} x$

**Ans.** (a) Given differential equation is,

$$\frac{dy}{dx} + (\tan x)\, y = \cos x,\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

IF $= e^{\int \tan x\, dx} = e^{\log \sec x} = \sec x$

$\therefore$ Complete solution, $y \,.\, (\text{IF}) = \int \cos x \,.\, (\text{IF})\, dx + C$

$y \,.\, \sec x = \int \cos x \,.\, \sec x\, dx + C$

$\Rightarrow y \,.\, \sec x = \int 1 dx + C = x + C$

$\Rightarrow y = (x + C) \cos x$

**27.** Consider the following LPP

Max $f = 5x + 12y$

Subject to constraints

$x + 5y \leq 50,\ 6x + 3y \leq 36,\ x \leq 5, x \geq 0, y \geq 0$

The number of extreme points of the feasible region are

(a) 4 (b) 5 (c) 6 (d) 7

**Ans.** (b) Given, LPP, Max $f = 5x + 12y$. Subject to constraints

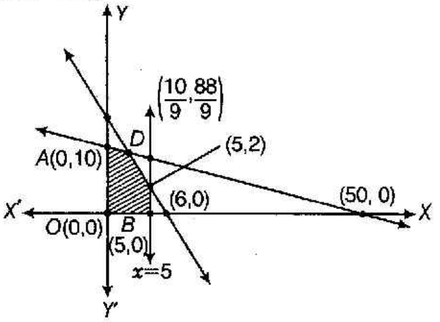
$x + 5y \leq 50,\ 6x + 3y \leq 36,\ x \leq 5,\ x \geq 0,\ y \geq 0$

First we convert all the inequalities into equality

| **Equation** | | **Points** |
|---|---|---|
| $x + 5y = 50$ | — | (50, 0) and (0, 10) |
| $6x + 3y = 36$ | — | (6, 0) and (0,12) |
| $x = 5$ | — | (5, 0) |

Now, plotting these points into the plotting paper.

Let (10 units = 1 sq)



$\therefore$ There are five extreme points $O(0, 0)$, $A(0, 10)$, $B(5, 0)$, $C(5, 2)$ and $D\left(\frac{10}{9}, \frac{88}{9}\right)$

**28.** Solution of the initial value problem $(2\cos y + 3x)\,dx - x\sin y\,dy = 0$, $y(1) = 0$ is

(a) $x^2\cos y - y^3 = 1$ (b) $x^2\sin y + y^3 = 0$
(c) $x^2\cos y + x^3 = 2$ (d) $y^2\sin x + y^3 = 0$

**Ans.** (c) Given differential equation is

$$(2\cos y + 3x)\,dx - x\sin y\,dy = 0,\ y(1) = 0$$

$$\Rightarrow\ (2x\cos y + 3x^2)\,dx - x^2\sin y\,dy = 0$$

$$\Rightarrow\ 2x\cos y\,dx - x^2\sin y\,dy + 3x^2\,dx = 0$$

$$\Rightarrow\ \int d(x^2\cos y) + \int 3x^2\,dx = 0 \text{ on integrating}$$

$$\Rightarrow\ x^2\cos y + 3.\frac{x^3}{3} = C \Rightarrow x^2\cos y + x^3 = C \qquad \text{...(i)}$$

$$\because\ y(1) = 0$$

$$\therefore\ (1)^2\cos 0 + (1)^3 = C$$

$$\Rightarrow\ 1 + 1 = C \Rightarrow C = 2$$

Now, from Eq. (i), we get $x^2\cos y + x^3 = 2$

**29.** A particular solution of the differential equation

$$\frac{d^5y}{dx^5} - 3\frac{d^4y}{dx^4} + 3\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} = 2e^x \text{ is}$$

(a) $\frac{1}{3}x^3e^x$ (b) $\frac{1}{2}x^3e^x$ (c) $\frac{1}{6}x^3e^x$ (d) $\frac{2}{3}x^3e^x$

**Ans.** (a) Given differential equation

$$\frac{d^5y}{dx^5} - 3\frac{d^4y}{dx^4} + 3\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} = 2e^x$$

$$\Rightarrow\ (D^5 - 3D^4 + 3D^3 - D^2)\,y = 2e^x \quad \left[\because D \equiv \frac{d}{dx}\right]$$

Auxiliary equation, $m^5 - 3m^4 + 3m^3 - m^2 = 0$

$$\Rightarrow\ m^2(m^3 - 3m^2 + 3m - 1) = 0$$

$$\Rightarrow\ m^2(m-1)(m^2 - 2m + 1) = 0$$

$$\Rightarrow\ m^2(m-1)(m-1)^2 = 0$$

$$\therefore\ m = 0, 0, 1, 1, 1$$

$$\therefore\ \text{PI} = \frac{2e^x}{D^2(D-1)^3} = \frac{2}{(1)^2}\frac{e^x}{(D-1)^3} = \frac{2}{1}.e^x\frac{1}{(D+1-1)^3}.1$$

$$= 2e^x\frac{1}{D^3}.1 = 2e^x.\iiint 1\,dx\,dx\,dx$$

$$= 2e^x\iint x\,dx\,dx$$

$$= 2e^x\int\frac{x^2}{2}\,dx = 2e^x\ \frac{x^3}{6} = \frac{e^xx^3}{3}$$

**30.** The volume of the tetrahedron bounded by the planes $z = 0$, $x = 0$, $y = 0$ and $y + z - x = 1$ is

(a) $\frac{1}{6}$ (b) 6
(c) 1 (d) $\frac{1}{3}$

**Ans.** (a) Given planes,

$$z = 0,\ x = 0,\ y = 0 \text{ and } y + z - x = 1$$

$\therefore$Required volume of the tetrahedron

$$= \int_{x=0}^{-1}\int_{y=0}^{1+x}\int_{z=0}^{1+x-y} dx\,dy\,dz$$

$$= \int_{x=0}^{-1}\int_{y=0}^{1+x}\int_{z=0}^{1+x-y} dz\,dy\,dx$$

$$= \int_{x=0}^{-1}\int_{y=0}^{1+x}(1 + x - y)\,dy\,dx$$

$$= \int_{x=0}^{-1}\left[\frac{-(1+x-y)^2}{2}\right]_0^{1+x} dx$$

$$= -\frac{1}{2}\int_{x=0}^{-1}[1 + x - 1 - x)^2 - (1+x)^2]\,dx$$

$$= \frac{1}{2} = \int_0^{-1}(1+x)^2\,dx = \frac{1}{2}\left[\frac{(1+x)^3}{3}\right]_0^{-1}$$

$$= \frac{1}{6}[(1-1)^3 - (1+0)^3] = \frac{1}{6}(0-1) = -\frac{1}{6}$$

$= \frac{1}{6}$ (neglecting negative sign)

**31.** The general solution of the non-homogeneous differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 12y = 150\cos 3x \text{ is}$$

(a) $c_1e^{-3x} + c_2e^{4x} - 7\cos 3x - \sin 3x$
(b) $c_1e^{3x} + c_2e^{-4x} - 7\cos 3x + \sin 3x$
(c) $c_1e^{3x} + c_2e^{-4x} + 7\cos 3x + \sin 3x$
(d) $c_1e^{3x} + c_2e^{-4x} - 7\cos 3x - \sin 3x$

**Ans.** (b) Given differential equation is

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 12y = 150\cos 3x$$

$$\therefore\ (D^2 + D - 12)\,y = 15\cos 3x$$

Auxiliary equation, $m^2 + m - 12 = 0$

$$\Rightarrow\ m^2 + 4m - 3m - 12 = 0$$

$$\Rightarrow\ (m+4)(m-3) = 0$$

$$\Rightarrow\ m = -4, 3$$

$$\therefore\ \text{CF} = c_2e^{-4x} + c_1e^{3x}$$

Now, $\text{PI} = \frac{150\cos 3x}{(D^2 + D - 12)} = \frac{150\cos 3x}{(-9 + D - 12)}$

$$= \frac{150}{(D-21)}\times\frac{(D+21)}{(D+21)}\times\cos 3x$$

$$= \frac{150(D+21)}{(D^2 - 441)}\times\cos 3x$$

$$= \frac{150\,(D+21)\cos 3x}{(-9-441)} = \frac{150\,(D+21)\cos 3x}{-450}$$

$$\Rightarrow -\frac{1}{3}\{D\cos 3x + 21\cos 3x\}$$

$$= -\frac{1}{3}\{-3\sin 3x + 21\cos 3x\} = \sin 3x - 7\cos 3x$$

$\therefore$ Required solution,

$$y = CF + PI$$

$$y = c_2e^{-4x} + c_1e^{3x} + \sin 3x - 7\cos 3x$$

or $c_1e^{3x} + c_2e^{-4x} - 7\cos 3x + \sin 3x$

**32.** What is the probability of getting an even number less than 5, in tossing a fair die?

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$

(c) $\frac{5}{6}$ (d) $\frac{1}{6}$

**Ans.** (b) Total sample space, $n(S) = 6$

Favourable events = {an even number less than 5} = {2, 4}

∴ Total favourable events $n(E) = 2$

∴ Required probability $= \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$

**33.** For the table

| x | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| f(x) | 1 | 2 | 9 | 28 |

the divided difference $f[1, 2, 3]$, is

(a) 6 (b) 13

(c) 3 (d) 1

**Ans.** (a) Given table

| x | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| f(x) | 1 | 2 | 9 | 28 |

From the table, we observe that, $f(x) = (x^3 + 1)$

$\because \quad f(0) = 1, f(1) = 2, f(2) = 9, f(3) = 27$

We have, $f(a, b) = \frac{f(b) - f(a)}{b - a} = \frac{b^3 + 1 - a^3 - 1}{b - a}$

$$= \frac{(b-a)(b^2 + ab + a^2)}{(b-a)}$$

$$= (a^2 + ab + b^2) \quad \text{...(i)}$$

Again, $f(a, b, c) = \frac{f(b, c) - f(a, b)}{c - a}$

$$= \frac{1}{c-a}[b^2 + bc + c^2 - (a^2 + ab + b^2)] \quad \text{[using Eq. (i)]}$$

$$= \frac{1}{c-a}[bc + c^2 - a^2 - ab]$$

$$= \frac{1}{c-a}[(c-a)(c+a) + b(c-a)]$$

$$= \frac{1}{(c-a)}.(c-a)[a + b + c] = (a + b + c)$$

∴ $f(1, 2, 3) = 1 + 2 + 3 = 6$

**34.** The Lagrange form of the interpolating polynomial that fits the data

| x | 0 | 1 | 2 |
|---|---|---|---|
| f(x) | 1 | 2 | 5 |

is

(a) $\frac{1}{2}(x - 1)(x - 2) - 2x(x - 2) + \frac{5}{2}x(x - 1)$

(b) $\frac{1}{2}(x - 1)(x - 2) + 2x(x + 2) + \frac{5}{2}x(x - 1)$

(c) $2(x - 1)(x - 2) + \frac{1}{2}x(x + 2) + \frac{2}{5}x(x - 1)$

(d) $2(x - 1)(x - 2) - \frac{1}{2}x(x + 2) + \frac{2}{5}x(x - 1)$

**Ans.** (a) Given that,

| x | 0 | 1 | 2 |
|---|---|---|---|
| f(x) | 1 | 2 | 5 |

Here, $x_0 = 0, x_1 = 1, x_2 = 2$

By Lagrange's formula, we have

$$f(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} f(x_0) + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} f(x_1) + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} f(x_2)$$

On substituting the values of $x_0, x_1, x_2$ in this, we get

$$f(x) = \frac{(x-1)(x-2)}{(0-1)(0-2)} \times 1 + \frac{(x-0)(x-2)}{(1-0)(1-2)} \times 2 + \frac{(x-0)(x-1)}{(2-0)(2-1)} \times 5$$

$$= \frac{1}{2}(x-1)(x-2) - 2x(x-2) + \frac{5}{2}(x-1)x$$

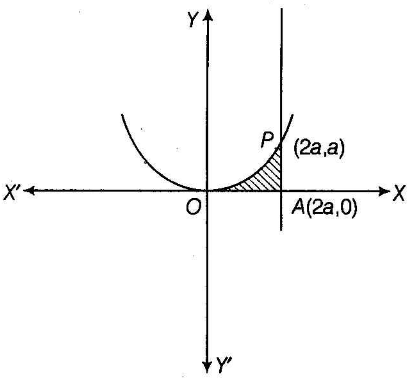
**35.** The area of the region enclosed by the parabola $x^2 = 4ay$ and the line $x = 2a$ with X-axis is

(a) $\frac{4}{3}a^2$ (b) $\frac{3}{2}a^2$

(c) $\frac{3}{4}a^2$ (d) $\frac{2}{3}a^2$

**Ans.** (d) Given curves,

$x^2 = 4ay$ and the line $x = 2a$



On solving both curve, we get

$(2a)^2 = 4ay \Rightarrow 4ay = 4a^2 \Rightarrow y = a$

So, the intersection point is $(2a, a)$.

∴Required area $= \int_0^{2a} y\, dx = \int_0^{2a} \frac{x^2}{4a} dx$

$$= \frac{1}{4a}\left[\frac{x^3}{3}\right]_0^{2a} = \frac{1}{12a} \times 8a^3 = \frac{2a^2}{3}$$

**36.** The integral $\int_{-1}^{1} f(x)\,dx$, where $f$ is continuous on $[-1, 1]$, is approximated by the formula

$$\int_{-1}^{1} f(x)\,dx \simeq \alpha f\left(-\frac{1}{\sqrt{2}}\right) + \beta f\left(\frac{1}{\sqrt{2}}\right)$$

Suppose the approximation is exact for all polynomials of degree $\leq 1$. Then, the value of $\alpha$ is

(a) $-1$ (b) $1$

(c) $\frac{1}{\sqrt{2}}$ (d) $-\frac{1}{\sqrt{2}}$

**Ans.** (b) Let the polynomial of degree $\leq 1$ is

$$f(x) = ax + b \qquad \text{...(i)}$$

Which is continuous on $[-1, 1]$.

Also given that,

$$\int_{-1}^{1} f(x)\,dx = \alpha f\left(-\frac{1}{\sqrt{2}}\right) + \beta f\left(\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \int_{-1}^{1} (ax+b)\,dx = \alpha\left\{-\frac{a}{\sqrt{2}} + b\right\} + \beta\left\{\frac{a}{\sqrt{2}} + b\right\}$$

$$\Rightarrow 2\int_{0}^{1} b\,dx = (\alpha+\beta)\,b + \left(\frac{-\alpha}{\sqrt{2}} = \frac{\beta}{\sqrt{2}}\right)a$$

$$\Rightarrow 0a + 2b = (\alpha+\beta)\,b + \left(-\frac{\alpha}{\sqrt{2}} + \frac{\beta}{\sqrt{2}}\right)a$$

On comparing, we get

$$\alpha + \beta = 2 \qquad \text{...(ii)}$$

and $-\frac{\alpha}{\sqrt{2}} + \frac{\beta}{\sqrt{2}} = 0$

$\therefore \quad \alpha = \beta$

From Eq. (ii), we get $\alpha = \beta = 1$

**37.** If (2, 1) is a critical point of $f(x, y)$ and $f_{xx}(2,1)$ $f_{yy}(2,1) - [f_{xy}(2,1)]^2 < 0$, then

(a) (2, 1) is a saddle point

(b) (2, 1) is a point of local maximum

(c) (2, 1) is a point of local minimum

(d) further investigation is required to determine the nature of the point

**Ans.** (a)

Here, $rt - s^2 < 0$

$\Rightarrow$ $f$ has neither maximum nor minimum at (2, 1).

$\Rightarrow$ (2, 1) is a saddle point.

**38.** If $f(x) = \int_{a}^{x^2} t(t-1)\,dt$, then

(a) $f$ has a local maximum at $x = 0$ and a local minimum at $x = 1$

(b) $f$ has local minima at $x = 0$ and $x = 1$

(c) $f$ has local maximum at $x = 1$ and a local minimum at $x = 0$

(d) $f$ has local maxima at $x = 1$ and $x = 0$

**Ans.** (a) $f'(x) = x^2(x^2-1) \cdot 2x = 2x^3(x^2-1)$

For maxima or minima

$$f'(x) = 0 \Rightarrow x = 0, -1, 1$$

$$f''(x) = 10x^4 - 6x^2$$

$$f''(0) = 0, f''(1) > 0$$

$\Rightarrow$ $f(x)$ has local minimum at $x = 1$.

$$f'''(x) = 40x^3 - 12x$$

$$f^{iv}(x) = 120x^2 - 12$$

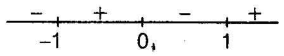
Clearly, $f'''(0) = 0$ and $f^{iv}(0) < 0$

$\therefore$ $x = 0$ is point of maximum of $f(x)$.

**Alternate method**

$$f'(x) = 2x^3(x^2-1)$$

Critical points are $x = -1, 0, 1$



Clearly, $f'(x)$ changes sign from + to – at $x = 0$ and – to + at $x = 1$.

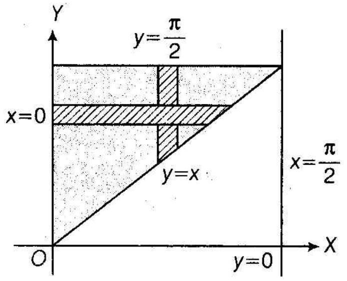
$\Rightarrow$ $f(x)$ has local maximum at $x = 0$ and minimum at $x = 1$.

**39.** The value of the integral $\int_{0}^{\frac{\pi}{2}} \int_{x}^{\frac{\pi}{2}} \frac{\sin y}{y}\,dy\,dx$, is

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) 2

**Ans.** (c) By changing the order of integration, we get



$$\int_{0}^{\pi/2}\int_{x}^{\pi/2} \frac{\sin y}{y}\,dy\,dx = \int_{0}^{\pi/2}\int_{0}^{y} \frac{\sin y}{y}\,dy\,dx$$

$$= \int_{0}^{\pi/2} \frac{\sin y}{y}\,dy \int_{0}^{y} dx = \int_{0}^{\pi/2} \sin y\,dy = (-\cos y)_{0}^{\pi/2}$$

$$= -\left(\cos\frac{\pi}{2} - \cos 0\right) = 1$$

**40.** The value of the arithmetic expression $-a * \frac{b}{c} + d\%\,k$f or integers variables

$$a = 5, b = 3, c = 2, d = 5, k = 3 \text{ is}$$

(a) $-5.5$ (b) $-5$

(c) $-3$ (d) $-1$

**Ans.** (b) $\therefore \; -a * b / c + d\%k$

$$= -5 * \frac{3}{2} + 5\%3 = -\frac{15}{2} + 2$$

$$= -7 + 2 = -5$$

**41.** Let $S_1 = \{2\}, S_2 = \{4, 6\}, S_3 = \{8, 10, 12\}$, $S_4 = \{14, 16, 18, 20\}$ and so on. The sum of elements of $S_{10}$ is

(a) 990 (b) 1000
(c) 1010 (d) 1020

**Ans.** (c) Let $S = 2 + 4 + 8 + 14 + \ldots + t_n$

Also, $S = 2 + 4 + 8 + \ldots + t_{n-1} + t_n$

On subtraction, we get

$$t_n = 2 + 2 + 4 + 6 + \ldots + \text{upto } n \text{ terms}$$
$$= 2 + [2 + 4 + 6 + \ldots + \text{upto } (n-1) \text{ terms}]$$
$$= 2 + 2 \cdot \frac{(n-1)\,n}{2} = 2 + n^2 - n$$

$\therefore \quad t_{10} = 100 - 10 + 2 = 92$

$\Rightarrow \quad S_{10} = \{92, 94, 96, \ldots\ldots \text{upto 10 terms}\}$

$\therefore$ Sum of elements

$$= 92 + 94 + 96 + \ldots + \text{upto 10 terms}$$
$$= \frac{10}{2}[2 \times 92 + 9 \times 2] = 1010$$

**42.** The locus of intersection of two lines $\sqrt{3}x - y = 4k\sqrt{3}$ and $k(\sqrt{3}x + y) = 4\sqrt{3}$ for different values of $k$ is a hyperbola. The eccentricity of the hyperbola is

(a) 1.5 (b) $\sqrt{3}$
(c) 2 (d) $\frac{\sqrt{3}}{2}$

**Ans.** (c) Given, lines are

$$\sqrt{3}x - y = 4k\sqrt{3} \qquad \ldots(\text{i})$$

and $k(\sqrt{3}x) + ky = 4\sqrt{3}$

$$\Rightarrow \quad \sqrt{3}x + y = \frac{4\sqrt{3}}{k} \qquad \ldots(\text{ii})$$

From Eqs. (i) and (ii), we get

$$3x^2 - y^2 = 48$$
$$\frac{x^2}{16} - \frac{y^2}{48} = 1$$

Which is a hyperbola.

$\therefore \quad a = 4, b = 4\sqrt{3}$

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{\frac{16 + 48}{16}} = \sqrt{\frac{64}{16}} = 2$$

**43.** If $x = 1$ is the directrix of the parabola $y^3 = kx - 8$, then $k$ is

(a) $\frac{1}{8}$ (b) 8 (c) 4 (d) $\frac{1}{4}$

**Ans.** (c) We have, $y^2 = kx - 8$

$$\Rightarrow \quad y^2 = k\left(x - \frac{8}{k}\right)$$

which is a form of $y^2 = 4AX$

So, $X = x - 8/k, A = k/4$

$\therefore$ Equation of directrix, $X = -A$

$$\Rightarrow \quad x - \frac{8}{k} = -\frac{k}{4}$$
$$\Rightarrow \quad x = \frac{8}{k} - \frac{k}{4}$$
$$\Rightarrow \quad 1 = \frac{8}{k} - \frac{k}{4}$$
$$\Rightarrow \quad 4k = 32 - k^2$$
$$\Rightarrow \quad k^2 + 4k - 32 = 0$$
$$\Rightarrow \quad k^2 + 8k - 4k - 32 = 0$$
$$\Rightarrow \quad (k + 8)(k - 4) = 0$$
$$k = 4, -8$$

**44.** If the foci of the ellipse $b^2x^2 + 16y^2 = 16b^2$ and the hyperbola $81x^2 - 144y^2 = \frac{81 \times 144}{25}$ coincide, then the value of $b$ is

(a) 1 (b) $\sqrt{5}$ (c) $\sqrt{7}$ (d) 3

**Ans.** (c) Since, foci coincide,

$$\therefore \quad 16 - b^2 = \frac{144}{25} + \frac{81}{25}$$
$$\Rightarrow \quad 16 - b^2 = \frac{225}{25}$$
$$\Rightarrow \quad b^2 = 16 - \frac{225}{25}$$
$$= \frac{400 - 225}{25} = \frac{175}{25} = 7$$
$$\therefore \quad b = \sqrt{7}$$

**45.** The condition that the line $lx + my + n = 0$ becomes a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, is

(a) $a^2l + b^2m + n = 0$ (b) $al^2 + bm^2 = n^2$
(c) $al + bm = n$ (d) $a^2l^2 + b^2m^2 = n^2$

**Ans.** (d) We have equation of line,

$$lx + my + n = 0$$

and $\quad y = \frac{-l}{m}x + \left(\frac{-n}{m}\right) \qquad \ldots(\text{i})$

We know that, if the line $y = mx + c$ touches the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Then, $c^2 = a^2m^2 + b^2$

$$\therefore \quad \frac{n^2}{m^2} = a^2\frac{l^2}{m^2} + b^2$$
$$\Rightarrow \quad n^2 = a^2l^2 + b^2m^2$$

**46.** The value of $\sin 20° \sin 40° \sin 80°$ is

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$
(c) $\frac{\sqrt{3}}{8}$ (d) $\frac{1}{8}$

**Ans.** (c) We have, $\sin 20° \sin 40° \sin 80°$

$$= \sin 20° \sin(60° - 20°) \sin(60° + 20°)$$

$= \sin 20° (\sin^2 60° - \sin^2 20°)$

$= \sin 20° \left[\frac{3}{4} - \sin^2 20°\right]$

$= \frac{1}{4} [3 \sin 20° - 4 \sin^2 20°]$

$= \frac{1}{4} \sin 3 \times 20° = \frac{1}{4} \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{8}$

**47.** Two non-negative numbers whose sum is 9 and the product of the one number and square of the other number is maximum, are

(a) 5 and 4 (b) 3 and 6
(c) 1 and 8 (d) 7 and 2

**Ans.** (b) Let two numbers be $x$ and $y$, where $x > 0$, $y > 0$.

Given, $x + y = 9$ ...(i)

and $z = x \cdot y^2$ ...(ii)

$\Rightarrow \quad z = x(9 - x)^2$

$= x(81 + x^2 - 18x) = x^3 - 18x^2 + 81x$

$\Rightarrow \quad \frac{dz}{dx} = 3x^2 - 36x + 81 = 3(x^2 - 12x + 27)$

$\because \quad \frac{dz}{dx} = 0$

$\Rightarrow \quad x^2 - 9x - 3x + 27 = 0$

$\Rightarrow \quad (x - 9)(x - 3) = 0$

$\Rightarrow \quad x = 3, \ x = 9$

$x = 3$ and $y = 6$ $\quad [\because x = 9$ not possible$]$

$\therefore$So, numbers are 3 and 6.

**48.** In $\Delta ABC$, if $a = 2$, $b = 4$ and $\angle C = 60°$, then $A$ and $B$ are respectively equal to

(a) 90°, 30° (b) 45°, 75° (c) 60°, 60° (d) 30°, 90°

**Ans.** (a) In $\Delta ABC$,

$a = 2, \ b = 4, \ \angle C = 60°$

$\because \quad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\because \quad \cos 60° = \frac{4 + 16 - c^2}{2 \times 2 \times 4}$

$\Rightarrow \quad \frac{1}{2} = \frac{20 - c^2}{16}$

$\Rightarrow \quad c^2 = 12 \quad \Rightarrow \quad c = 2\sqrt{3}$

Now, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\Rightarrow \quad \frac{\sin A}{a} = \frac{\sin C}{c}$

$\Rightarrow \quad \frac{\sin A}{2} = \frac{\sin 60°}{2\sqrt{3}} \quad \Rightarrow \quad \sin A = \frac{1}{2}$

$\Rightarrow \quad \angle A = 30°$

Now $\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad 30° + \angle B + 60° = 180°$

$\Rightarrow \quad \angle B = 90°$

$\therefore \quad \angle A = 30°$ and $\angle B = 90°$

**49.** If $\int \frac{xe^x}{\sqrt{1+e^x}} dx = f(x)\sqrt{1+e^x} - 2 \log \frac{\sqrt{1+e^x}}{\sqrt{1+e^x}} + C$, then $f(x)$ is

(a) 2x − 1 (b) 2x − 4 (c) x + 4 (d) x − 4

**Ans.** (b) Let $I = \int \frac{xe^x}{\sqrt{1 + e^x}} dx$

Put $1 + e^x = t^2 \Rightarrow e^x \, dx = 2t \, dt$

$x = \log(t^2 - 1)$

$\therefore \quad I = 2 \int \log \frac{(t^2 - 1)}{t} t \, dt = 2 \int \log(t^2 - 1) \, dt$

$= 2\left[t \log(t^2 - 1) - 2 \int \frac{t^2}{t^2 - 1} dt\right]$

$= 2\left[t \log(t^2 - 1) - 2dt \int \left(1 + \frac{1}{t^2 - 1}\right) dt\right]$

$= 2\left[t \log(t^2 - 1) - 2t - \log\left(\frac{t - 1}{t + 1}\right)\right] + C$

$= 2x\sqrt{1 + e^x} - 4\sqrt{1 + e^x} - 2\log\left(\frac{\sqrt{1 + e^x} - 1}{\sqrt{1 + e^x} + 1}\right) + C$

$= (2x - 4)\sqrt{1 + e^x} - 2\log\left(\frac{\sqrt{1 + e^x} - 1}{\sqrt{1 + e^x} + 1}\right) + C$

Hence, $f(x) = 2x - 4$

**50.** The average marks of boys in a class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is

(a) 80% (b) 60% (c) 40% (d) 20%

**Ans.** (a) Given, combined average of class is 50. Let number of boys in class be $x$ and number of girls in class by $y$.

By combined average formula, $50 = \frac{52 \times x + 42 \times y}{x + y}$

$\Rightarrow \quad 50x + 50y = 52x + 42y$

$\Rightarrow \quad 8y = 2x$

$\Rightarrow \quad \frac{x}{y} = \frac{4}{1}$

$\Rightarrow \quad \frac{x}{x + y} = \frac{4}{5}$

Hence, ratio of boys to total number of students is $\frac{4}{5}$ and

percentage $= \frac{4}{5} \times 100 = 80\%$

# ENGLISH LANGUAGE AND COMPREHENSION

**Directions** *(Q. No. 51-52) Answer the following questions based on the given paragraph.*

The fossil remains of the first flying vertebrates, the pterosaurs, have intrigued palaecontologists for more than two centuries. How such large creatures, which weighted in some cases as much as a piloted hang glider and had wingspans from 8 to 12 m, solved the problems of powered flight, and exactly what these creatures were-reptiles or birds are among the questions scientists have puzzled over.

Perhaps the least controversial assertion about the pterosaurs is that they were reptiles. Their skulls, pelvises, and hind feet are reptilian. The anatomy of their wings suggests that they did not evolve into the class of birds. In Pterosaurs a greatly elongated fourth finger of each forelimb supported a wing like membrane. The other fingers were short and reptilian, with sharp claws. In birds, the second finger is the principle strut of the wing, which consists primarily of feathers. If the Pterosaur walked or remained stationary, the fourth finger and with it the wing, could only turn upward in an extended inverted V-shape along side of the animal's body.

The pterosaurs resembled both birds and bats in their overall structure and proportions. This is not surprising because the design of any flying vertebrate is subject to aerodynamic constraints. Both the Pterosaurs and the birds have hollow bones a feature that represents a saving in weight. In the birds, however, these bones are reinforced more massively by internal struts.

**51.** It can be inferred from the passage that the scientists now generally agree that

(a) enormous wingspan of the pterosaurs enable them to fly great distances.
(b) structure of the skeleton of the pterosaurs suggests a close evolutionary relationship to bats.
(c) fossil remains of the pterosaurs reveal how they solved the problem of powered flight.
(d) pterosaur were reptiles.

**Ans.** (d) Pterosaurs are reptiles is generally agreeable statement.

**52.** According to the passage, the skeleton of pterosaurs can be distinguished from that of a bird by the

(a) size of its wingspan.
(b) presence of hollow spaces in its bones.
(c) anatomic origin of its wing strut.
(d) presence of hook like projections on its hind feet.

**Ans.** (c) Anatomic origin of wing strut is the main point to distinguish between bird or non-bird.

**53.** The following passage consists of six sentences. The first sentence ($S_1$) is given in the beginning. The final sentence ($S_6$) is given in the last. The middle four sentences are jumbled up and labelled as P, Q, R and S. You are required to find out the proper sequence of the four sentences and mark accordingly.

S1: Unlike many modern thinkers, Tagore had no blueprint for the world's salvation.
P : His thought will therefore never be out of data.
Q : He merely emphasised certain basic truths which men may ignore only at their peril.
R : He believed in no particularism.
S : He was what Gandhiji rightly termed the great sentinel.
$S_6$ : As a poet he will always delight, as a singer he will always enchant, as a teacher he will always enlighten.

The proper sequence should be

(a) SRPQ (b) PRQS
(c) RSPQ (d) RQPS

**Ans.** (d) RQPS is the correct consequence or order of sentences.

**Directions** *(Q. Nos. 54-55) Each question consists of a word printed in capital letters, followed by four words or phrases. Choose the word or phrase that is most similar in meaning to the word in capital letters.*

**54.** EXASPERATE

(a) Pacify (b) Mollify
(c) Irritate (d) Placate

**Ans.** (a) **Exasperate** to make someone extremely annoyed and impatient.

Its similar meaning **Irritate**.

**55.** INIMICAL

(a) Antagonistic (b) Anonymous
(c) Fanciful (d) Accurate

**Ans.** (a) **Inimical** unfriendly. Its similar meaning.

**Antagonistic** disliking someone or behaving in a very unfriendly way.

**56.** Which of the underlined parts in the sentence given below is a mistake which may need to be deleted or modified?

He can be able to pass the test in flying colours without any difficulties whatsoever.

(a) be able to (b) flying colours
(c) difficulties (d) whatsoever

**Ans.** (a) Correct sentence: He can pass the test in flying colours without any difficulties whatsoever.

**57.** The idiom 'I will be a monkey's uncle' means

(a) to want to keep a monkey
(b) that I have been enlightened
(c) that I have been fooled
(d) to express disbelief

**Ans.** (c) I will be monkey's uncle mean that I have been fooled.

**58.** Fill in the blanks.

I could not .......... him to attend the meeting.

(a) prevail over
(b) prevail upon
(c) prevail about
(d) prevail in

**Ans.** (b) **Prevail upon** to ask or persuade someone to do something.

**Directions** *(Q. Nos. 59-60) Each question consists of a word printed in capital letters, followed by four words or phrases. Choose the word or phrase that is most nearly opposite in meaning to the word in capital letters.*

**59.** OPPROBRIUM

(a) Honour
(b) Prudence
(c) Ostentation
(d) Umbrage

**Ans.** (a) **Opprobrium** very strong criticism of something that you do not approve of or dishonour.
Its opposite-**Honour** to respect.

**60.** INCESSANT

(a) Perpetual
(b) Persistent
(c) Sporadic
(d) Unrelenting

**Ans.** (c) **Incessant** continuing for a long time without stopping in a way that is annoying.
Its opposite.

**Sporadic** not regular or frequent.

# COMPUTER AWARENESS

**61.** What one of the following statements is always true?

(a) A compiled program uses more memory than an interpreted program.
(b) A compiler converts a program to a lower level language for execution.
(c) A compiler for a high level language takes less memory than its interpreter.
(d) Compiled programs take more time to execute than interpreted programs.

**Ans.** (b) A compiler converts a high level program into low level language (Machine language) for execution.

**62.** The capacity of a memory unit is defined by the number of works multiplied by the number of bits per word. How many separate address and data line are needed for a memory of $4K \times 16$?

(a) 10 address lines and 16 data lines
(b) 12 address lines and 10 data lines
(c) 12 address lines and 16 data lines
(d) 12 address lines and 8 data lines

**Ans.** (c) $4K \times 16 = 2^{12} \times 16$

$\Rightarrow$ We should have 12 address lines and 16 data lines.

**63.** The main disadvantage of direct mapping of cache organisation is that

(a) it doesn't allow simultaneous access to the intended data and its tag.
(b) it is more expensive that other type of organisations.
(c) the cache hit ratio is degreaded if two or more blocks used alternatively map into the same block frame in the cache.
(d) the number of blocks required for the cache increases linearly with the size of the main memory.

**Ans.** (d) The number of blocks required for the cache increases linearly with the size of the main memory which is the main disadvantage of direct mapping of cache organisation.

**64.** The first instruction of bootstrap loader program of an operating system is stored in

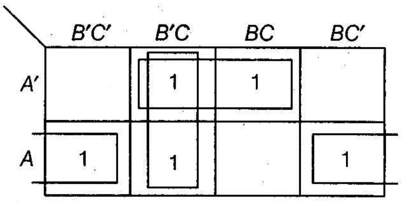
(a) RAM
(b) BIOS
(c) Hard disk
(d) None of these

**Ans.** (b) BIOS = Basic Input Output System which stores first instruction of boots trap loader program.

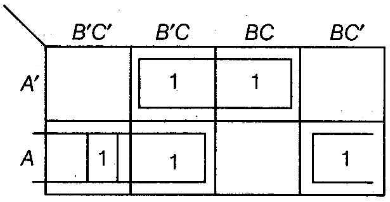
**65.** The function $AB'C + A'BC + ABC' + A'B'C$ is equivalent to

(a) $AC' + AB + A'C$
(b) $AB' + AC' + A'C$
(c) $A'B + AC' + AB'$
(d) $A'B + AC + AB'$

**Ans.** (b) It will be simplified by Karnaugh map as follows. There are three pairs.



$\Rightarrow$ Function is $AC' + B'C + A'C$



**66.** The addition of 4 bit, 2's complement binary numbers 1101 and 0100 results in

(a) 0001 and an overflow (b) 1001 and no overflow

(c) 0001 and no overflow (d) 1001 and an overflow

**Ans.** (c) 1's complement of 1101 = 0010

2's complement of 1101 = 0010 + 1 = 0011

and 1's complement of 0100 = 1011

2's complement of 0100 = 1011 + 1 = 1100

$$\begin{array}{r} 0011 \\ +1100 \\ \hline 1111 \end{array}$$

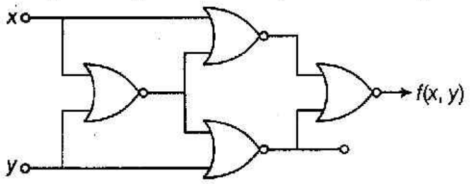
Addition both 4 bits:

It's one's complement = 0000

and 2's complement = 0000 + 1 = 0001

There is not overflow.

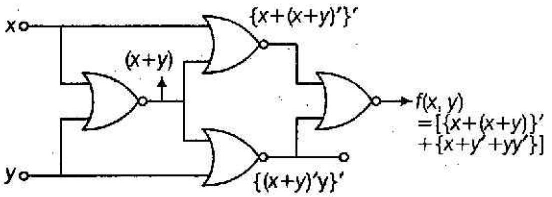
**67.** Identify the logic function performed by the circuit.



(a) Exclusive OR (b) Exclusive NOR

(c) NAND (d) NOR

**Ans.** (b)



$$f(x,y) = [(x + x' \cdot y')' + \{x' \cdot y' + y\}']'$$

(by De-Morgan's law

$= [x' \cdot (x' \cdot y') + (x' \cdot y') \cdot y']'$ (by De-Morgan's law)

$= \{x' \cdot (x+y) + (x+y) \cdot y'\}'$ (by De-Morgan's law)

$= \{x' \cdot (x+y)\}' \cdot \{(x+y) \cdot y\}'$ (by De-Morgan's law)

$= [x + (x+y)'] \cdot [(x+y)' + y]$ (by De-Morgan's law)

$= [x + x' \cdot y'] \cdot [x' \cdot y' + y]$ (by De-Morgan's law)

$= (x + x')(x + y') \cdot (x' + y) \cdot (y' + y)$ (by Distributive law)

$= 1 \cdot (x + y') \cdot (x' + y) \cdot 1$ (by Complement law)

$= (x + y)' \cdot (x' + y)$ (by Identity law)

$= x \cdot x' + xy + y'x' + y'y$ (by Distributive law)

$= 0 + xy + y'x' + 0$ (by Complement law)

$= xy + x'y'$ (by Identify and Commutative law)

which represent exclusive NOR.

**68.** Which of the following is (are) true about virtual memory systems that uses pages?

I. The virtual address space can be larger than the amount of physical memory.

II. Programs must be resident in main memory throughout their execution.

III. Pages correspond to semantic characteristics of the programs.

(a) I only (b) II only

(c) I and II (d) I and III

**Ans.** (c) Virtual memory systems uses pages since, the virtual address space can be larger than the amount of physical memory and pages corresponds to semantic characteristics of the programs.

**69.** How many bits are required to store an ASCII character?

(a) 7 (b) 6

(c) 8 (d) None of these

**Ans.** (a) In ASCII, there are 7 bits required are store.

**70.** What is the output of a JK flip-flop during next clock cycle, when $J = 1$, $K = 1$? Assume, Q is the output during the current clock cycle.

(a) 1 (b) 0 (c) Q (d) Q′

**Ans.** (d) $J = 1$, $K = 1$ produces complementary output.

Since, Q is the output during the current clock cycle $\overline{Q}$ (or $Q'$) will be the output of next clock cycle.

**71.** What are the values of the variables, **i**, **j** and **k** after execution of the following program segment?

int **i** = 1, **j** = 2, **k** = 3; **i** + = **j** + = **k**;

(a) **i** = 3, **j** = 5, **k** = 6 (b) **i** = 3, **j** = 6, **k** = 5

(c) **i** = 6, **j** = 3, **k** = 5 (d) **i** = 6, **j** = 5, **k** = 3

**Ans.** (d) int **i** = 1, **j** = 2, **k** = 3, **i** + = **j** + = **k**,

we know that, every expression in 'C' language compiled from right to left.

**Step Ist** $K = 3$

**Step IInd j += k**

⇒ **j** = **j** + **k** = 2 + 3 = 5 [∵ **j** = 2]

**Step IIIrd i + = j**

⇒ **i** = **i** + **j** = 1 + 5 = 6 [∵ **j** = 5]

**72.** Let $X$ and $Y$ be 4 bit registers with initial contents as 1011 and 1001, respectively. The following sequence of operations are performed on the two registers

$Y \leftarrow X \oplus Y$

$X \leftarrow X \oplus Y$

$Y \leftarrow X \oplus Y$

Where ⊕ denotes *XOR* operation. The final contents of the two registers are

(a) $X = 1001$, $Y = 1011$ (b) $X = 101$, $Y = 1001$

(c) $X = 1011$, $Y = 1011$ (d) $X = 1001$, $Y = 1001$

**Ans.** (a) $X = 1011$

$Y = 1001$

$Y \leftarrow X \oplus Y \Rightarrow Y = 0010$

$X \leftarrow X \oplus Y$

$\Rightarrow X = 1001 \quad Y \leftarrow X \oplus Y$

$\Rightarrow Y = 1011$

$\Rightarrow X = 1001, Y = 1011$

**73.** An I/O processor controls the flow of information between

(a) cache memory and I/O devices
(b) main memory and I/O devices
(c) two I/O devices (d) cache and main memories

**Ans.** (b) An I/O processor controls the flow of information between main memory and I/O devices.

**74.** Which of the following devices will take highest time in taking the backup of the data from a computer?

(a) Magnetic disk (b) Pen drive
(c) CD (d) Magnetic tape

**Ans.** (d) Magnetic tape will take highest time in taking the backup of the data from a computer.

**75.** The errors that can be pointed out by compilers are

(a) syntax errors (b) semantic errors
(c) logical errors (d) internal errors

**Ans.** (a) The errors that can be pointed out by compilers are syntax errors.

**76.** $(2FAOC)_{16}$ is equivalent to

(a) $(195084)_0$
(b) $(0010111110100000 1100)_2$
(c) Both (a) and (b)
(d) None of the above

**Ans.** (c) From option (b),

Binary form 0010 1111 1010 0000 1100
Hexadecimal 2 F A Q C
$\therefore \quad (2FAOC)_{16} = (00101111101000001100)_2$
From option (a),
$(195084)_{10} = (00101111101000001100)_2 = (2FAOC)_{16}$

**77.** If the integer needs two bytes of storage, then maximum value of an unsigned integer is

(a) $2^{16} - 1$ (b) $2^{15} - 1$ (c) $2^{16}$ (d) $2^{15}$

**Ans.** (a) If the integer needs two bytes of storage, then maximum value of an unsigned integer is $2^{16} - 1$.

**78.** The minimum number of temporary variables needed to swap the contents of two variables is

(a) 1 (b) 2 (c) 3 (d) 0

**Ans.** (a) The minimum number of temporary variables needed to swap the contents of two variables is 1.

e.g. int x =10, y=10, z;

```
main ()
{
z = x;
x = y;
y = z;
}
```

**79.** Which of the following terms could be used to describe the concurrent processing of computer programs via CRTs, on one computer system?

(a) Time sharing
(b) Online processing
(c) Interactive processing
(d) All of the above

**Ans.** (c) Interactive processing could be used to describe the concurrent processing of computer programs via CRT's on one computer system.

**80.** Which of the following would most likely not be a symptom of a virus?

(a) Existing program files and icons disappear
(b) The CD-ROM stops functioning
(c) The web browser opens to an unusual home page
(d) Odd messages or images are displayed on the screen

**Ans.** (b) The CD-ROM stops functioning would most likely not be a system of a virus.

# LOGICAL AND ANALYTICAL REASONING

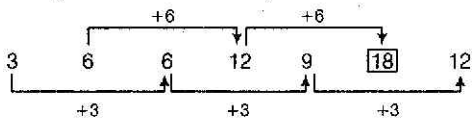
**81.** The missing number in the given series

3, 6, 6, 12, 9, ..., 12 is

(a) 15 (b) 18 (c) 11 (d) 13

**Ans.** (b) Given series, 3, 6, 6, 12, 9, ..., 12

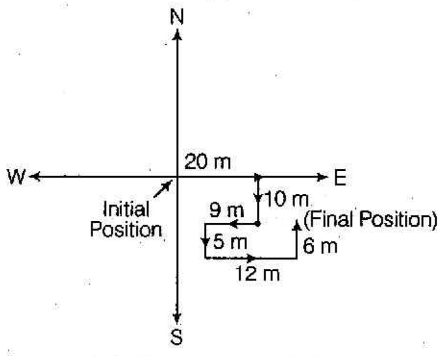
Split the given series into two parts



**82.** A man runs 20 m towards east and turns right, runs 10 m and turns right, runs 9 m and turns left, runs 5 m and turns left, runs 12 m and finally turns left and runs 6 m. Which direction is the man facing?

(a) North (b) South
(c) East (d) West

**Ans.** (a)



Hence, the man is facing in the North Direction.

**Directions** (Q. Nos. 83-85) *Read the following passage carefully and answer the questions.*

Six boys A, B, C, D, E and F are marching in a line. They are arranged according to their heights, the tallest being at the back and the shortest in the front. F is between B and A. E is shorter than D but taller than C who is taller than A. E and F have two boys between them. A is not the shortest among them.

**83.** Where is E?
(a) Between A and B
(b) Between C and A
(c) Between D and C
(d) In front of C

**84.** If we start counting from the shortest, which boy is fourth in the line?
(a) E (b) A (c) D (d) C

**85.** Who is next to the shortest?
(a) C
(b) B
(c) E
(d) F

**Sol** (Q. Nos. 83-85)

According ot the condition,

$$B < F < A < C < E < D$$

(Shortest) (Tallest)

83. (c) E is between D and C

84. (d) C is fourth in the line.

85. (d) F is next to the shortest.

**86.** The letters P, Q, R, S, T, U and V not necessarily in that order represent seven consecutive integers from 22 to 33 and

1. U is as much less than Q as R is greater than S.
2. V is greater than U and Q.
3. Q is the middle term.
4. P is greater than S.

Then, the sequence of letters from the lowest value to the highest value, is
(a) TVPQRSU
(b) TRSQUPV
(c) TUSQRPV
(d) TVPQSRU

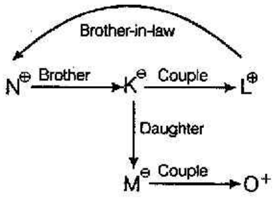
**Ans.** (c) By given condition, we get the required order (sequence) of letters from the lowest value to the highest value is

$$T < U < S < Q < R < P < V$$

i.e. TUSQRPV

**87.** Five persons K, L, M, N and O are sitting around a dining table. K is the mother of M, M is actually the wife of O, N is the brother of K and L is the husband of K. How is N related to L?
(a) Son
(b) Cousin
(c) Brother
(d) Brother-in-law

**Ans.** (d)



Hence, N is the Brother-in-law of L.

**88.** If 'ROAST' is coded as 'PQYUR' in a certain language, then 'SLOPPY' is coded in that language as
(a) MRNAQN
(b) NRMNQA
(c) QNMRNA
(d) RANNMQ

**Ans.** (c)

$R \xrightarrow{-2} P$ $S \xrightarrow{-2} Q$
$O \xrightarrow{+2} Q$ $L \xrightarrow{+2} N$
$A \xrightarrow{-2} Y$ $O \xrightarrow{-2} M$
$S \xrightarrow{+2} U$ $P \xrightarrow{+2} R$
$T \xrightarrow{-2} R$ $P \xrightarrow{-2} N$
$Y \xrightarrow{+2} A$

**89.** If 'leli broon' means 'yellow hat', 'pleka froti' means 'flower garden' and 'froti mix' means 'garden salad', then which word could mean 'yellow flower'?
(a) leli froti
(b) leli pleka
(c) plekafroti
(d) froti broon

**Ans.** (b) leli broon ⟶ yellow hat
pleka froti ⟶ flower garden
froti mix ⟶ garden salad
∴ flower ⟶ pleka
yellow ⟶ leli or broon

By option,
yellow flower ⟶ lelil pleka

**90.** If + is *, − is +, * is/and/is −, then $6 - 9 + 8 * \frac{3}{20}$ is equal to
(a) − 2 (b) 6 (c) 10 (d) 12

**Ans.** (c) $E = 6 - 9 + 8 * \frac{3}{20}$

By given condition,

$$E = 6 + 9 * \frac{8}{3} - 20$$
$$E = 6 + 3 * 8 - 20$$
$$E = 6 + 24 - 20$$
$$E = 10$$

**91.** In a certain year, there were exactly four Fridays and four Mondays in January. On what day of the week did the 20th of January fall that year?
(a) Saturday
(b) Sunday
(c) Thursday
(d) Tuesday

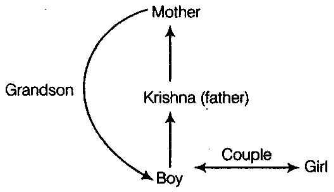
**Ans.** (b) Let in a month of January.

(4 times) Friday ⟶ 25, 18, 11, 4 (dates)
(4 times) Monday ⟶ 28, 21, 14, 7 (dates)
Then, required dates of Sunday,
Sunday ⟶ 27, 20, 13, 6
So, 20th January fell on Sunday.

**92.** Krishna said, "This girls is the wife of grandson of my mother." How is Krishna related to girl?

(a) Father
(b) Father-in-law
(c) Husband
(d) Grandfather

**Ans.** (b)



Krishna is "father-in-law" of that girl.

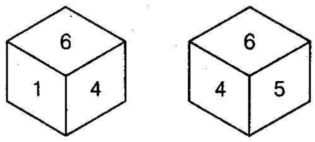
**93.** A study of native born residents in a area of Adivasis found that two-third of the children developed considerable levels of nearsightedness after starting school, while their illiterate parents and grandparents, who had no opportunity for formal schooling, showed no signs of this disability.

If the above statements are true, which of the following conclusions is most strongly supported by them?

(a) Only people who have the opportunity for formal schooling develop nearsightedness
(b) People who are illiterate do not suffer from nearsightedness
(c) The nearsightedness in the children is caused by the visual stress required by reading and other class work
(d) Only literate people are nearsighted

**Ans.** (c) From the statements, we clearly say that the reason behind the nearsightedness of the children is caused by the visual stress required by reading and other class work.

**94.** Two positions of a dice are shown below. When number 1 is on the top, what number will be at the bottom?



(a) 2 (b) 3
(c) 5
(d) Cannot be determined

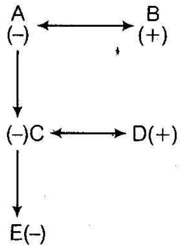
**Ans.** (c) After observation of given two dice, we get the number 5 is at the bottom of the dice, when number 1 is on the top.

**95.** There is a family party consisting of two fathers, two mothers, two sons, one father-in-law, one mother-in-law, one daughter-in-law, one grandfather, one grandmother and one grandson.

What is the minimum number of persons required, so that this is possible?

(a) 5 (b) 6
(c) 7 (d) 8

**Ans.** (a) Let '–' means 'male' and '+' means 'female'.



Two fathers (A, C) Two mothers (B, D)
Two sons (C, E) One father-in-law (A)
One mother-in-law (B) One daughter-in-law (D)
One grandfather (A) One grandmother (B)
Hence, there are 5 minimum number of persons. One Grandson (E).

**96.** Each word in parenthesis below is formed in a method. This method is used in all four examples.

SNIP (NICE) PACE
TEAR (EAST) FAST
TRAY (RARE) FIRE
POUT (OURS) CARS

Based on this method, the word in the parenthesis of CANE (?) BATS is

(a) NEAT (b) CATS (c) ANTS (d) NETS

**Ans.** (c) SNIP (NICE) PACE
TEAR (EAST) FAST
POUT (OURS) CARS
∴ CANE (AN + TS) BATS
Hence, ? = ANTS

**97.** 'College' is related to 'student' in the same way as Hospital is related to

(a) Doctor
(b) Nurse
(c) Medicine
(d) Patient

**Ans.** (d) In the college education is given to students, in the same way treatment is given to the 'Patient' in 'Hospital'.

**Directions** (Q. Nos. 98-100) *Read the following passage carefully and answer the questions.*

Five houses lettered A, B, C, D and E are built in a row next to each other. The houses are lined up in the order A, B, C, D and E. Each of the five houses have coloured roofs and chimneys. The roof and chimney of each house must be painted as follows.

1. The roof must be painted either green, red or yellow.
2. The chimney must be painted either white, black or red.
3. No house may have the same colour chimney as the colour of roof.
4. No house may use any of the same colours that adjacent house uses.
5. House E has a green roof.
6. House B has a red roof and a black chimney.

**98.** Which of the following is true?

(a) Atleast two houses have black chimney
(b) Atleast two houses have red roofs
(c) Atleast two houses have white chimneys.
(d) Atleast two houses have green roofs

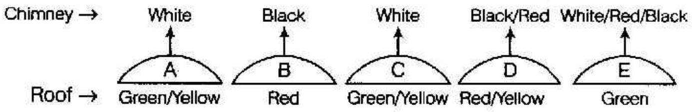
**99.** If house C has a yellow roof, then which of the following must be true?

(a) House E has a white chimney
(b) House E has a black chimney
(c) House E has a red chimney
(d) House D has a red chimney

**100.** What is the maximum number of green roofs?

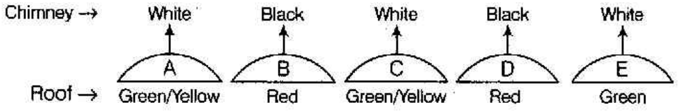
(a) 1 (b) 2 (c) 3 (d) 4

**Sol** (Q. Nos. 98-100)



**98.** (c) Atlest two houses have white chimney is true.

**99.** (a)



**100.** (c) The maximum number of green roofs are 3.